New Geiger-Nuttall law for alpha decay of heavy nuclei

Zhongzhou Ren and Yuejiao Ren

Department of Physics, Nanjing University, Nanjing 210093, China

Introduction

The Geiger-Nuttall (GN) law is a famous formula written in many text books of modern physics and nuclear physics. It relates alpha-decay half-lives to decay energies as $\log_{10}T_{1/2}=aQ^{-1/2}+b$, where *a* and *b* are the given constants for even-even nuclei on an isotopic chain. Based on this law, the Viola-Seaborg (VS) formula is presented,

$$\log_{10}T_{1/2}=(aZ+b)Q^{-1/2}+cZ+d+h,$$

where *a, b, c*, and *d* are determined from alpha-decay of even-even nuclei and *h* is the blocking factor for odd nucleons. It has been widely used to systematize the data of alpha decay and to predict the half-lives of unknown nuclei.

In the last decade, as result of the special interest in heavy and superheavy nuclei, the interest in alpha-decay systematics is continuing and the interest in cluster radioactivity is increasing. In 2004, Ren *et al.* proposed the new formula for half-lives of cluster radioactivity,

$$\log_{10}T_{1/2}=aZ_1Z_2Q^{-1/2}+cZ_1Z_2+d+h,$$

where Z_1 and Z_2 are the atomic number of daughter nuclei and clusters. This formula can be considered as a natural generalization of the VS formula from simple alpha decay to complex cluster radioactivity. In view of the same nature of alpha decay and cluster radioactivity, we also deduced the unified formula of half-lives for them from the WKB barrier penetration probability,

$$\log_{10}T_{1/2}=a\mu^{1/2}Z_1Z_2Q^{-1/2}+c\mu^{1/2}(Z_1Z_2)^{1/2}+d+h,$$

where μ is the reduced mass of the cluster-daughter system. This formula can at the same time describe the complicated processes of alpha decay and cluster radioactivity effectively.

With the alpha-decay data of ground-state and high-spin isomers accumulating, it is interesting to see whether the relationship between half-lives and decay energies deviates systematically from the original GN law. By analyzing the behavior of systematic deviation, some quantum-mechanical effects can be considered as well. Then further extensions towards hindered alpha transitions and alpha-decaying isomers can be made.

Methods

When the unified formula of alpha decay and cluster radioactivity is used to calculated the half-lives of alpha transitions with the neutron number stepping over the *N=126* shell closure, a strong effect is observed. A dramatic deviation occurs for *N*≤126 nuclei on the isotopic chains of *Z=84-92*. It is shown in Fig. 1 for Po isotopes. To overcome this problem, some basic observables such as quantum numbers need to be included in the formula.

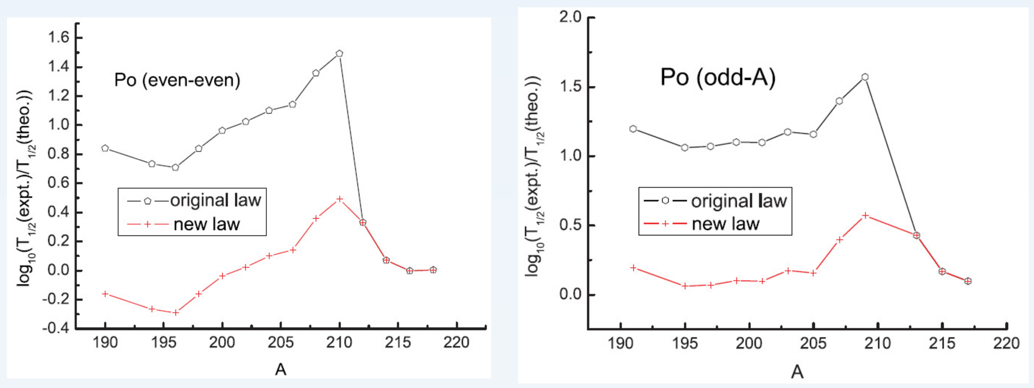

Fig 1. Ratios between experiment and theory for Po nuclei with the original law and with the new law. A sudden change across the *N=126* shell closure is seen with the original law.

The Schrödinger equation for the system is given as

$$\left\{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}-\left[V_N(r)+V_C(r)+\frac{l(l+1)\hbar^2}{2\mu r^2}\right]\right\}u_{nlj}(r)=E\,u_{nlj}(r)$$

The quantum numbers of alpha cluster are obtained by the Wildermuth rule, *G=2n+l*. When the neutron number *N* goes across the *N=126* shell closure (from *N*<126 to *N*>126), it is expected that the change of the global quantum number is ΔG=2. Therefore we introduce a quantum number *S*=- Δ*G*/2 to mock up the effect of the quantum numbers on half-lives. We define the values of *S* as follows: *S=0* for *N*≥128 and *S=1* for *N*≤126.

To include the effect of angular momentum and parity of alpha cluster on half-lives, we add a new term *Pl(l+1)*. This term can be approximately derived based on the WKB barrier penetration when the centrifugal potential is taken into account. To this end, the new GN law derived from the quantum tunneling theory can be written as

$$\log_{10}T_{1/2}=a\sqrt{\mu}Z_1Z_2Q^{-1/2}+b\sqrt{\mu}\sqrt{Z_1Z_2}+c+S+Pl(l+1)$$

Results

With the new GN formula, we firstly calculate the half-lives of the ground state of even-even nuclei with Z=84-92 and with Z=60-74. The numerical results are listed in Tables I and II. One can see that the calculated results generally agree with the experimental data within a factor of 2, showing the good reliability of the new GN formula.

Table I. The logarithm of alpha-decay half-lives of even-even Z=84-92 isotopes calculated with the new NG law and the corresponding experimental ones. The experimental *Q* values are also listed.

Nuclei	Q (MeV)	$\log_{10}T_{\rm expt}$ (s)	$\log_{10}T_{\rm theo}$ (s)	Nuclei	Q (MeV)	$\log_{10}T_{\rm expt}$ (s)	$\log_{10}T_{\rm theo}$ (s)
^{218}Po	6.115	2.27	2.27	^{218}Ra	8.546	-4.59	-4.46
^{216}Po	6.906	-0.84	-0.84	^{216}Ra	9.526	-6.74	-6.93
^{214}Po	7.833	-3.78	-3.86	^{214}Ra	7.273	0.39	0.45
^{212}Po	8.954	-6.52	-6.86	^{206}Ra	7.415	-0.62	-0.05
^{210}Po	5.407	7.08	6.59	^{204}Ra	7.636	-1.22	-0.78
^{208}Po	5.215	7.96	7.60	^{202}Ra	8.020	-2.58	-1.97
^{206}Po	5.327	7.14	7.00	^{232}Th	4.082	17.65	17.56
^{204}Po	5.485	6.28	6.18	^{230}Th	4.770	12.38	12.38
^{202}Po	5.701	5.15	5.12	^{228}Th	5.520	7.78	7.88
^{200}Po	5.981	3.79	3.83	^{226}Th	6.451	3.26	3.43
^{198}Po	6.309	2.27	2.43	^{224}Th	7.298	0.02	0.15
^{196}Po	6.657	0.77	1.06	^{222}Th	8.127	-2.69	-2.56
^{194}Po	6.987	-0.41	-0.14	^{220}Th	8.953	-5.01	-4.87
^{190}Po	7.693	-2.61	-2.45	^{218}Th	9.849	-6.96	-7.04
^{222}Rn	5.590	5.52	5.61	^{216}Th	8.071	-1.57	-1.39
^{220}Rn	6.405	1.75	1.91	^{214}Th	7.826	-1.00	-0.63
^{218}Rn	7.263	-1.46	-1.29	^{212}Th	7.952	-1.44	-1.03
^{216}Rn	8.200	-4.35	-4.20	^{238}U	4.270	17.15	17.17
^{214}Rn	9.208	-6.57	-6.82	^{236}U	4.573	14.87	14.84
^{212}Rn	6.385	3.16	2.99	^{234}U	4.858	12.89	12.85
^{210}Rn	6.159	3.95	3.94	^{232}U	5.414	9.34	9.44
^{208}Rn	6.261	3.37	3.50	^{230}U	5.993	6.25	6.40
^{206}Rn	6.384	2.74	2.99	^{228}U	6.803	2.74	2.82
^{204}Rn	6.546	2.01	2.33	^{226}U	7.701	-0.57	-0.47
^{226}Ra	4.871	10.70	10.67	^{224}U	8.620	-3.03	-3.29
^{224}Ra	5.789	5.50	5.56	^{222}U	9.500	-5.85	-5.59
^{222}Ra	6.679	1.58	1.65	^{218}U	8.786	-2.22	-2.75
^{220}Ra	7.592	-1.75	-1.62				

Table II. Same as in Table I, but for even-even nuclei with *Z=60-74*.

Nuclei	Q (MeV)	$\log_{10}T_{\rm expt}$ (s)	$\log_{10}T_{\rm theo}$ (s)	Nuclei	Q (MeV)	$\log_{10}T_{\rm expt}$ (s)	$\log_{10}T_{\rm theo}$ (s)
^{168}W	4.506	6.20	6.50	^{156}Er	3.487	9.84	10.04
^{166}W	4.856	4.74	4.53	^{154}Er	4.2799	4.68	4.61
^{164}W	5.2785	2.22	2.41	^{152}Er	4.9344	1.06	1.15
^{162}W	5.6773	0.48	0.64	^{154}Dy	2.946	13.98	13.56
^{160}W	6.065	-0.99	-0.91	^{152}Dy	3.726	6.93	7.04
^{158}W	6.613	-2.86	-2.86	^{150}Dy	4.3513	3.08	3.13
^{162}Hf	4.417	5.69	5.95	^{152}Gd	2.203	21.53	21.09
^{160}Hf	4.9024	3.29	3.28	^{150}Gd	2.808	13.75	13.56
^{158}Hf	5.4047	0.80	0.90	^{148}Gd	3.27121	9.37	9.26
^{156}Hf	6.028	-1.62	-1.62	^{148}Sm	1.9861	23.34	22.81
^{158}Yb	4.172	6.63	6.36	^{146}Sm	2.5284	15.51	15.17
^{156}Yb	4.811	2.42	2.75	^{144}Nd	1.9052	22.86	22.40
^{154}Yb	5.4742	-0.35	-0.31				

Next, we extend the calculations to the case of odd-*A* nuclei and the case of isomers. In Fig. 1, the deviation between calculated results and experimental data is shown for favored transitions of odd-*A* Po isotopes. It is clearly seen that the results with the new GN law agree with experimental data better. Note that ^{211}Po is not included in the figure because it is a hindered transition. Hindered transitions are very complicated, but the new GN law with the *l(l+1)* term proves to be successful in describing them. The results of hindered transitions are shown in Table III. They contain isomeric transitions from a 18^+ state and from 8^+ state in ^{212}Po and isotonic sequence *N=127* with *l=5* and odd parity.

Table III. The alpha-decay half-lives of two kinds of hindered transition with the new NG law: isomeric transitions in ^{212}Po and those from *N=127* isotones. The spin-parity involving in the transition are listed, together with the angular momentum of alpha particle.

Nuclei	Transition	ℓ	Q (MeV)	$T_{\rm expt}$ (s)	$T_{\rm theo}$ (s)
^{212}Po^n	$18^+\to0^+$	18	11.884	45.13	38.60
^{212}Po^m	$8^+\to0^+$	8	10.431	4.07×10^{-8}	8.69×10^{-8}
^{211}Po	$9/2^+\to1/2^-$	5	7.595	0.516	0.243
^{213}Rn	$9/2^+\to1/2^-$	5	8.243	0.0195	0.0157
^{215}Ra	$9/2^+\to1/2^-$	5	8.864	1.55×10^{-3}	1.63×10^{-3}
^{217}Th	$9/2^+\to1/2^-$	5	9.433	2.40×10^{-4}	2.81×10^{-4}
^{219}U	$9/2^+\to1/2^-$	5	9.860	5.5×10^{-5}	11.9×10^{-5}

Conclusion

The new GN law for the calculations of alpha-decay half-lives is proposed where the effects of quantum numbers are naturally taken into account. By including the change of the node number of the cluster wave function, the available data of even-even emitters both N≤126 and N≥128 are well reproduced. The inclusion of the term depending on the parity and angular momentum leads to a reliable description of the hindered transitions from *N=127* odd-*A* isotones and from high-spin isomers in ^{212}Po. The results of this work point to the simplicity of the underlying mechanism of the decay.

Reference

Yuejiao Ren and Zhongzhou Ren, Phys. Rev. C 85, 044608 (2012) and references therein.