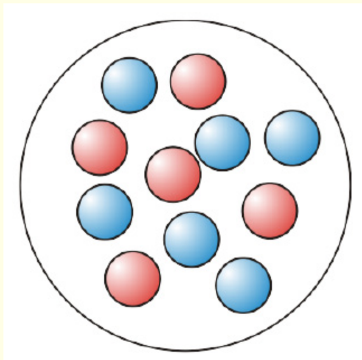
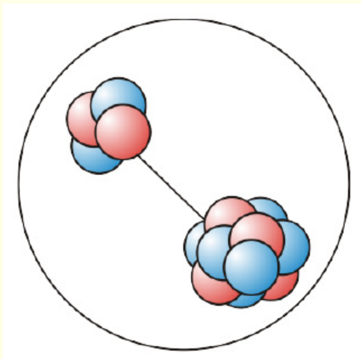
# Coupled-channels study of fine structure in the α decay of well-deformed nuclei



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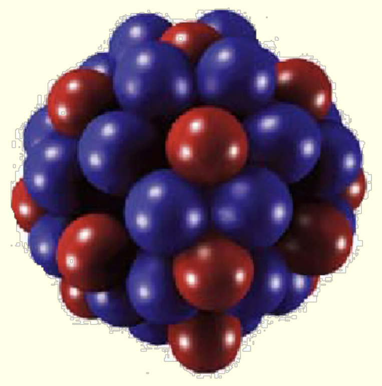

## Introduction

Alpha decay has proven to be a powerful and very precise tool to observe exotic nuclei and investigate their structure. Identification and knowledge of new synthesized elements and nuclides mainly resort to observing alpha-decay chains. For deformed emitters, low-lying states in a daughter nucleus are closely distributed so that they are all accessible to alpha transitions. And there is significant mixing of these channels during the tunneling. This is called fine structure.

Theoretically, alpha decay has been traditionally treated as a one-dimensional semi-classical problem even for deformed nuclei. This is apparently not consistent with the nature of alpha decay, i.e., three-dimensional quantum tunneling effect. Therefore, it is of the utmost importance to develop an exact quantum mechanics approach, where three-dimensional Schrödinger equations are exactly solved for quasibound states (this state is much less documented than bound and scattering states in textbooks).

## Multi-channel cluster model

I. The dynamic picture we consider here is shown in Fig. 1. The coupled-channel Schrödinger equations are written as

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2}-\frac{l_\alpha(l_\alpha+1)}{r^2}\right)-(Q_0-E_I)\right]u_\alpha(r)+\sum_{\alpha'}V_{\alpha\alpha'}(r)u_{\alpha'}(r)=0,\ [\alpha\equiv(nlI)].$$

(1) The multipole expansion is employed to deal with the interaction matrix, including the dynamic effect of the core,

$$V_{\alpha,\alpha'}(r)=\sum_\lambda \upsilon_\lambda(r)(-1)^\lambda\sqrt{(2l'+1)(2I+1)(2\lambda+1)/4\pi}$$
$$\times\langle l'\lambda 00|l0\rangle W(l'\lambda JI;lI')\langle\Phi_I\|\Omega_\lambda\|\Phi_{I'}\rangle$$

(2) Considering the Pauli exclusion principle, the channel quantum numbers are determined by the Wildermuth rule, which links the shell model with the cluster model.

(3) In contrast to exponent attenuation of bound states at large distances, the channel wave functions show an oscillatory behavior like outgoing Coulomb-Hankel waves, as shown in Fig. 2. This is the remarkable difference between quasibound and bound states.

II. The structure part of alpha decay can be evaluated by a preformation factor. The internal structure of daughter states is considered by the hypothesis of the Boltzmann distribution (BD), as Einstein did for molecules with a set of discrete states in 1917.

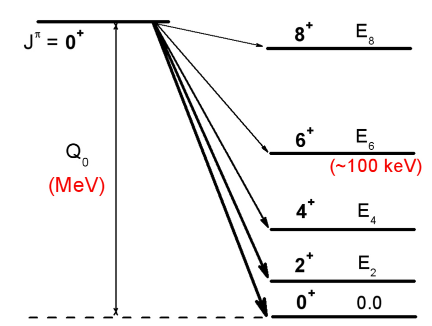


**Figure 1**. Schematic alpha decay for deformed even-even nuclei, including five decay channels.

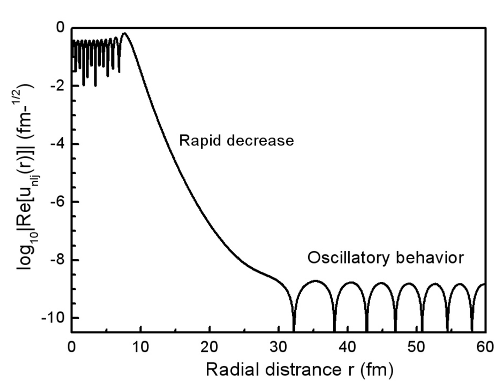


**Figure 2**. Real part of quasibound wave functions, showing oscillatory behavior at large distances.

## Results and discussion

First, let us discern the sensitivity of the results to the structure properties of daughter nuclei. The strong dependence of the branching ratios (BR) on the excitation energy of daughter states is evident in our calculations.

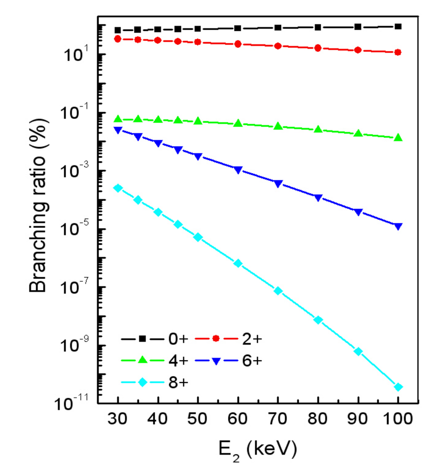


Fig. 3 illustrates the calculated BR as a function of the $E_{2^+}$ value in the alpha-decay of $^{244}$Cm. The decreasing trend of BR is more significant as we proceed to higher-spin states. But the variation of half-life is quite smooth as a function of the $E_{2^+}$ value.

**Figure 3. Sensitivity of the calculated BR to the energy spectrum of daughter nuclei for the emitter $^{244}$Cm.**

To check for the sensitivity of the results to nuclear deformation, we have calculated the results with various β values. The strong changes in BR to excited $4^+$ and $6^+$ states are shown in Fig. (4a). For the half-life, as shown in Fig. (4b), minor changes are found as a function of the β value.

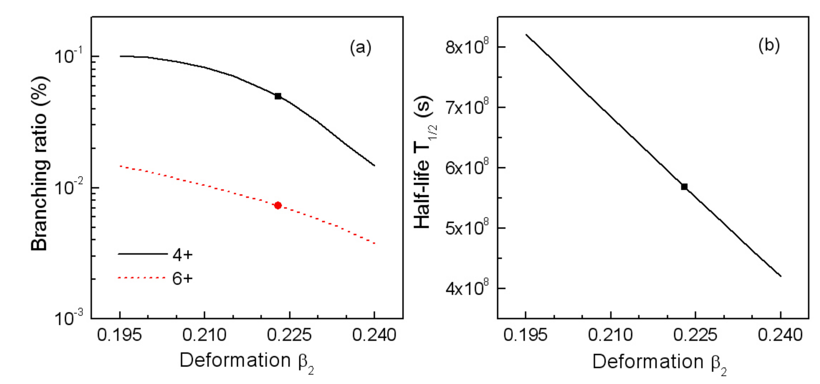


**Figure 4. Sensitivity of the calculated BR and half-life to the deformation βvalue for the emitter $^{244}$Cm.**

In conclusion, the results presented above show BR to high-spin states is an important and sensitive probe of the structure properties of daughter nuclei. According to this, there is a good chance to experimentally investigate nuclear structure properties. Besides, we would like to point out that our calculations have weak dependence on the model quantities, such as the number of channels, the expansion order, and the global quantum number, showing the good stability of the model. For the length limit, here we do not display it.

Next, let us discuss the calculations of alpha-decay half-lives. In spit of large deformation involved, the theoretical alpha-decay half-lives agree well with the experimental ones over a wide range of magnitude, from $10^{-3}$ to $10^{17}$ s. Fig. 5 shows the deviations of the calculated half-lives from the experimental data as a function of the mass number $A$ of the parent nuclei.

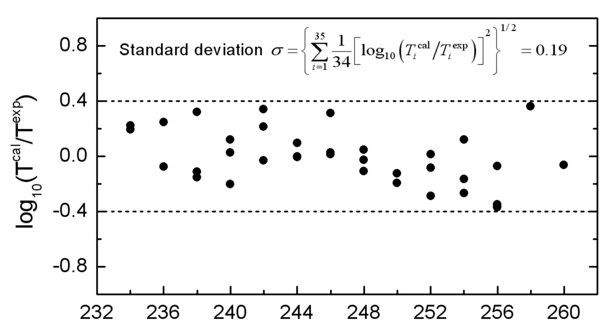


**Figure 5. Comparison of experimental half-lives with theoretical ones for 35 well-deformed emitters.**

Now, we transfer our attention to BR observed in alpha decay. Fig. 6 shows the comparison of the measured and calculated BR. For comparison calculations without the BD hypothesis are also given. One can see that, with the BD hypothesis, the points of all kind gather toward the line more evidently than without it. Furthermore, there exists good agreement between experiment and theory in Fig. (6a) . On the whole, the present calculations well reproduce the available experimental BR to $0^+$, $2^+$, $4^+$, $6^+$, and $8^+$ states with mean factors of 1.1, 1.3, 2.7, 3.3, and 3.2, respectively.

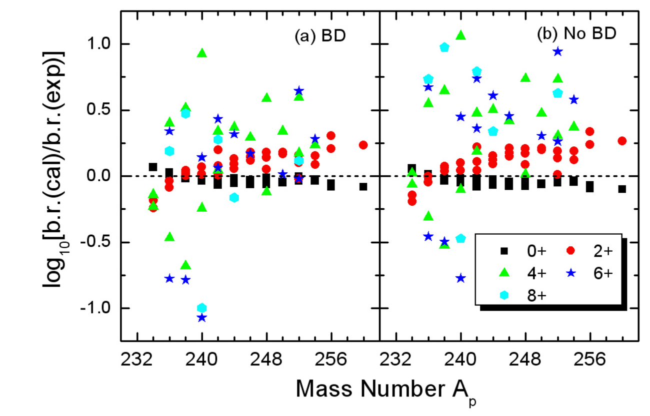


**Figure 6. Comparison of the calculated and experimental BR as a function of the parent mass number. Calculations with the hypothesis of the Boltzmann distribution and without it are, respectively, shown in the left and right panels.**

Table I shows predicted BR of various approaches for the decay chain $^{246}$Cf--$^{242}$Cm--$^{238}$Pu, for which the available experimental data are relatively full. The other results calculated from (i) the coupled-channel approach with a parabolic repulsive core, (ii) the deformed WKB approach, (iii) the simple WKB barrier penetration approach are given in the last three columns. For the transitions to high-spin states, the differences are striking. The WKB calculations have a tendency toward overestimating BR to excited $4^+$ states by one order of magnitude, while the coupled-channel results give a good description of this quantity. This success could be important to interpret future observations for heavy nuclei and even superheavy nuclei.

**Table I. Comparison of the calculated BR with the experimental data and other theoretical results for the decay chain $^{246}$Cf--$^{242}$Cm-- $^{238}$Pu. The focus is on the transitions to high-spin states**

| | Transition | BR (exp) | BR[a] (cal) | BR (i) | BR (ii) | BR[a] (iii) |
|---|---|---|---|---|---|---|
| $^{246}$Cf: | $0^+\to0^+$ | 79.3 | 70.8 | 61.7 | 55.9 | 73.9 |
| | $0^+\to2^+$ | 20.6 | 28.9 | 38.1 | 33.0 | 24.3 |
| | $0^+\to4^+$ | 0.15 | 0.29 | 0.19 | 9.79 | 1.82 |
| | $0^+\to6^+$ | 0.016 | 0.024 | – | 1.35 | 0.032 |
| | $0^+\to8^+$ | – | $1.5\times10^{-4}$ | – | – | – |
| $^{242}$Cm: | $0^+\to0^+$ | 74.08 | 68.87 | 63.30 | 59.0 | 76.00 |
| | $0^+\to2^+$ | 25.92 | 31.04 | 36.60 | 32.3 | 22.70 |
| | $0^+\to4^+$ | 0.035 | 0.077 | 0.101 | 7.85 | 1.333 |
| | $0^+\to6^+$ | 0.0046 | 0.0053 | – | 0.82 | 0.0152 |
| | $0^+\to8^+$ | $2.0\times10^{-5}$ | $3.8\times10^{-5}$ | – | 0.0345 | $3.2\times10^{-5}$ |
| $^{238}$Pu: | $0^+\to0^+$ | 70.91 | 67.63 | 64.96 | 62.00 | 77.41 |
| | $0^+\to2^+$ | 28.98 | 32.02 | 34.97 | 31.20 | 21.52 |
| | $0^+\to4^+$ | 0.105 | 0.343 | 0.069 | 6.28 | 1.06 |
| | $0^+\to6^+$ | 0.0030 | 0.0005 | – | 0.492 | 0.009 |
| | $0^+\to8^+$ | $6.8\times10^{-6}$ | $2.03\times10^{-5}$ | – | 0.0144 | $1.41\times10^{-5}$ |

[a] The BD hypothesis is employed for various daughter states.

## Conclusions

Based on the coupled-channel Schrödinger equation, we formulate a theoretical model for the alpha decay of well-deformed even-even nuclei, namely, the multi-channel cluster model (MCCM).

In contrast to the traditional alpha-decay theories where BR to high-spin states are hard to understand, alpha-decay half-lives and fine structures observed in alpha decay are well described by the five-channel microscopic calculations. It is also found that the alpha transition to high-spin states is a powerful tool to probe the energy spectrum and deformation of daughter nuclei.

The predictive power of this study may be a useful tool for estimating the alpha-decay properties of superheavy nuclei to be measured.

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## For further information


Please contact zren@nju.edu.cn （任中洲）. More information on this work and related subjects can be found in the following papers :



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