Microscopic level density for nucleons and deformed core

-- The ”best” microscopic basis for **combinatorial level density**: Folded Yukawa potential plus various residual effects – pairing and surface vibrations

-- realistic with respect to nuclear excitations

-- schematic treatment of angular momentum

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-- Distinction between spherical and deformed nuclei:

-- Spherical nuclei: Bethe formula (1937)

-- Deformed nuclei: Ericsson formula (1960)

-- **Many-particle plus rotor level density**:

-- very schematic with respect to nuclear excitations

-- precise coupling of angular momentum

-- ”rotational enhancement” is carried by deformed core – continous transition from Bethe to Ericsson formula

Combinatorial level density

based on:

Folded Yukawa single particle potential – well tested for nuclear Masses, single particle states, deformations, fission properties … (Möller and Nix)

-- basic: many-particle-many hole states
-- also: - pairing with many-quasiparticle BCS on each state
- surface vibrations with Tamm-Dankoff – avoid double counting
- configuration mixing

M or K ?

Quantum numbers: **additive**: E,N,Z,K,(π) angular momentum **projection**

external axis for spherical **intrinsic axis** for deformed

Bethe and Ericsson formulas

$$\rho\,(E,N,Z,I,\pi) \;=\; \rho_{proj}(E,N,Z,M,=I,\pi) \;-\; \rho_{proj}(E,N,Z,M=I+1,\pi)$$

Bethe formula for spherical nuclei

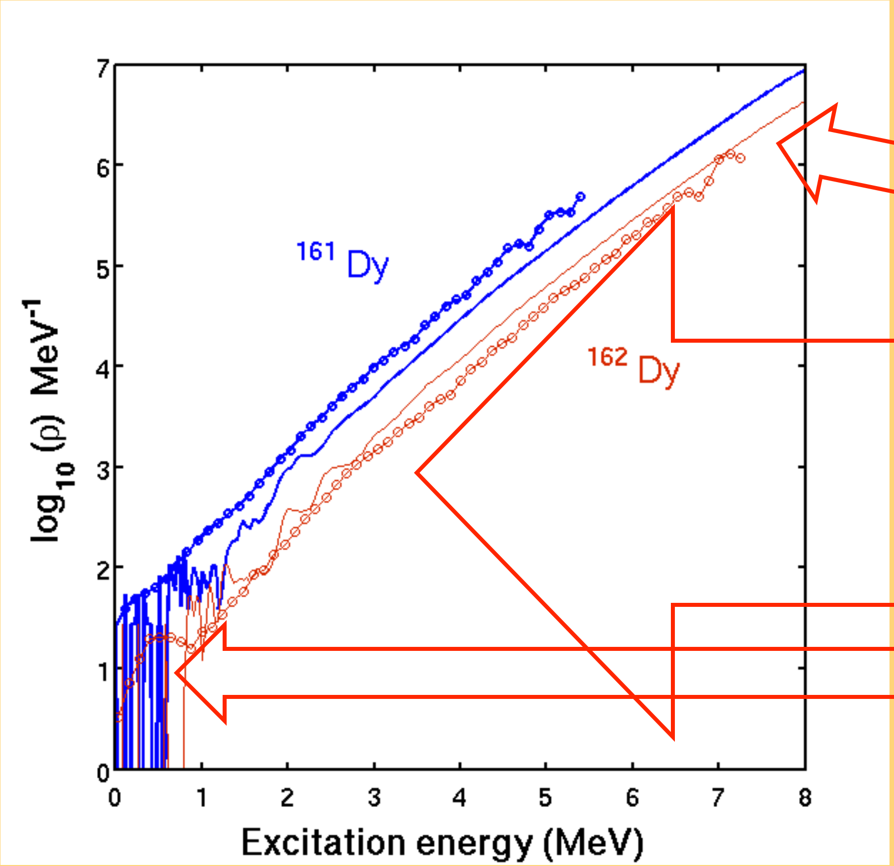
$$\rho\,(E,N,Z,I,\pi) \;=\; 1/2 \sum_{K=-I}^{K=I} \rho_{proj}(E-E_{rot}(I,K),N,Z,K,\pi)$$

$$E_{rot}(I,K) = \frac{I(I+1) - K^2}{2\,I}$$

Ericsson formula for deformed nuclei (simplified here)

Due to subtraction: Ericsson formula yields a much higher level density than Bethe formula --- deformation enhancement (rotational enhancement)

Example: ^{161,162}Dy



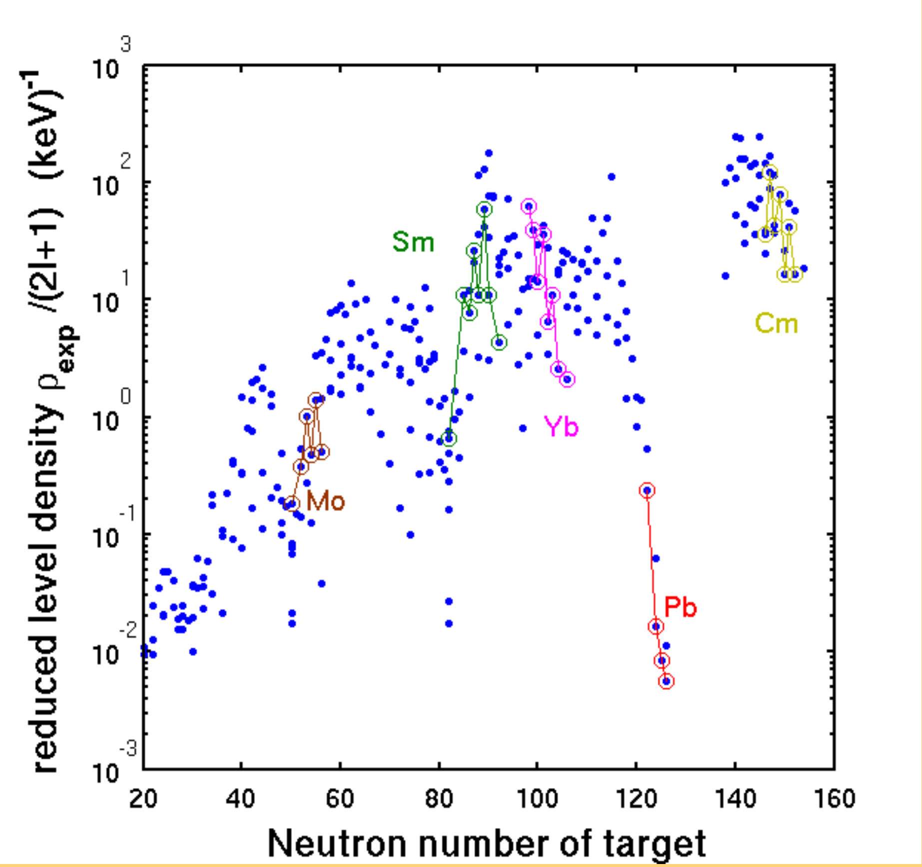
Data: **Oslo method**

Level count at neutron separation energy

Gamma spectra following transfer or inelastic reactions

level count at low excitation energy

Golden data: Level density at neutron separation energy



levels counted directly in small interval around 8 MeV of excitation energy

Smooth behavior of level density: Fermi Gas

$$\rho(A,E) = \frac{1}{\sqrt{48}}\frac{1}{E}\exp\left(2\sqrt{aE}\right) \tag{1}$$

$$\rho(N,Z,E) = \frac{\sqrt{\pi}}{12}\frac{1}{(aE)^{1/4}}\frac{1}{E}\exp\left(2\sqrt{aE}\right) \tag{2}$$

$$\rho(N,Z,E,K) = \frac{1}{12}\left(\frac{\hbar^2}{2\mathcal{J}}\right)^{1/2}\left[E-\frac{\hbar^2}{2\mathcal{J}}K^2\right]^{-3/2}\exp\left(2\sqrt{a(E-\frac{\hbar^2}{2\mathcal{J}}K^2)}\right) \tag{3}$$

$$\rho_{sph}(N,Z,E,I) =$$
$$\frac{2I+1}{12}\sqrt{a}\left(\frac{\hbar^2}{2\mathcal{J}}\right)^{3/2}\left[E-\frac{\hbar^2}{2\mathcal{J}}I(I+1)\right]^{-2}\exp\left(2\sqrt{a\left(E-\frac{\hbar^2}{2\mathcal{J}}I(I+1)\right)}\right) \tag{4}$$

$$\rho_{deformed}(N,Z,E,I,\pi) \approx$$
$$\frac{2I+1}{48}\left(\frac{\hbar^2}{2\mathcal{J}}\right)^{1/2}\left[E-\frac{\hbar^2}{2\mathcal{J}_\perp}I(I+1)\right]^{-3/2}\exp\left(2\sqrt{a\left(E-\frac{\hbar^2}{2\mathcal{J}_\perp}I(I+1)\right)}\right) \tag{5}$$

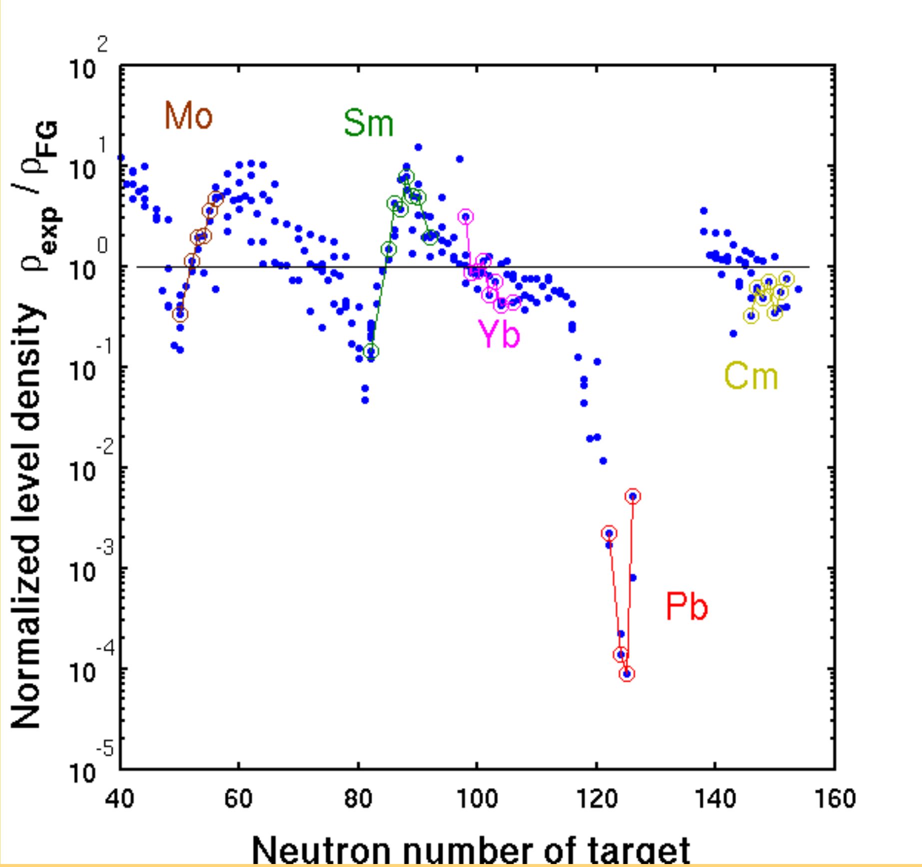
$$E \rightarrow \begin{cases} E-2\Delta & \text{for even-even nuclei} \\ E-\Delta & \text{for odd-A nuclei} \\ E & \text{for odd-odd nuclei} \end{cases}$$

level density parameter **a**

Spherical nuclei: Bethe formula

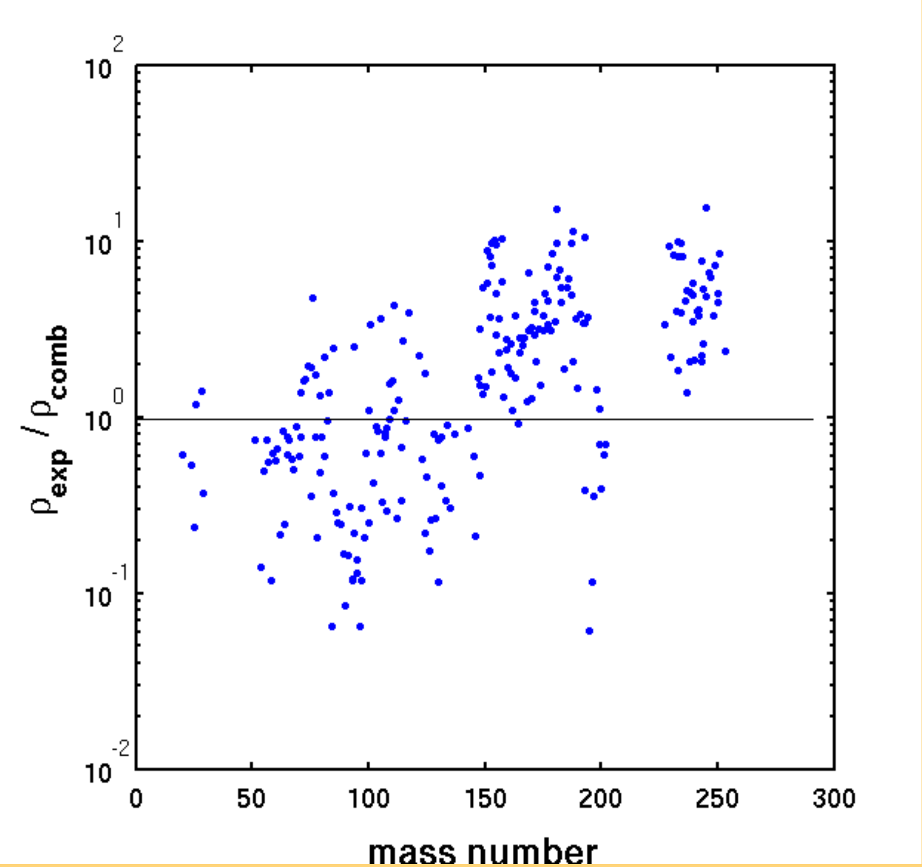
Deformed nuclei: Ericsson formula

Level density normalized by Fermi Gas level density



$$a = \frac{A}{11.3}$$

Experimental level density relative to combinatorial for deformed nuclei

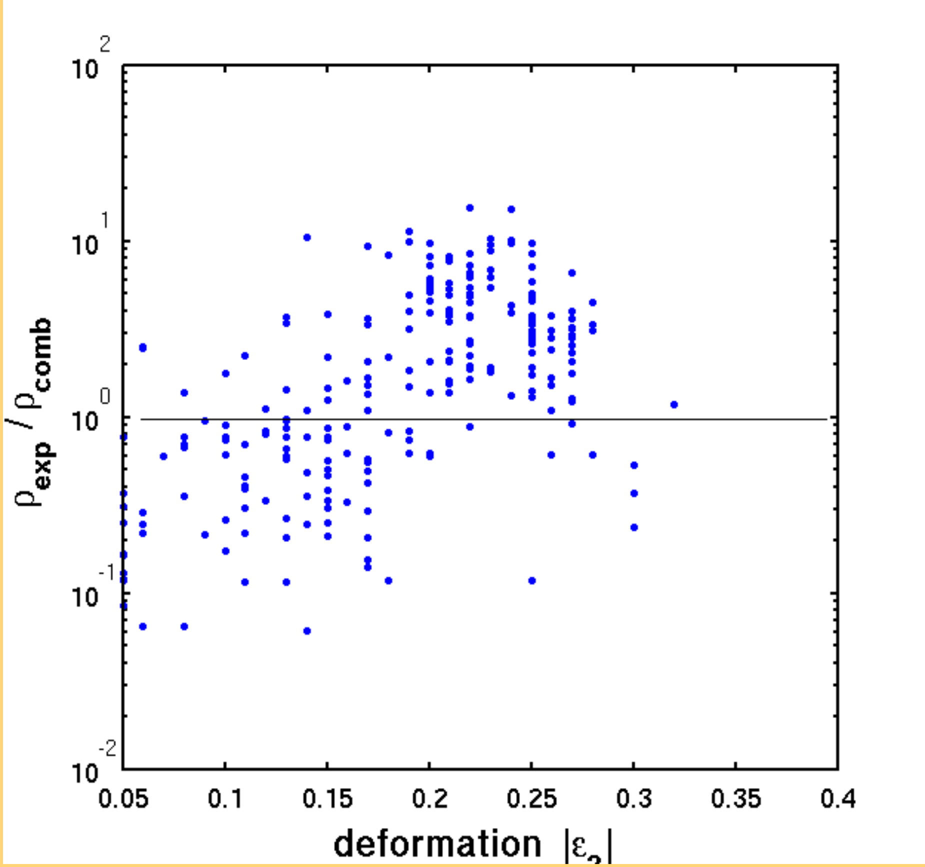


ρ_{comb}

underestimates level density for heavy nuclei

$f_{rms} = 3.7$

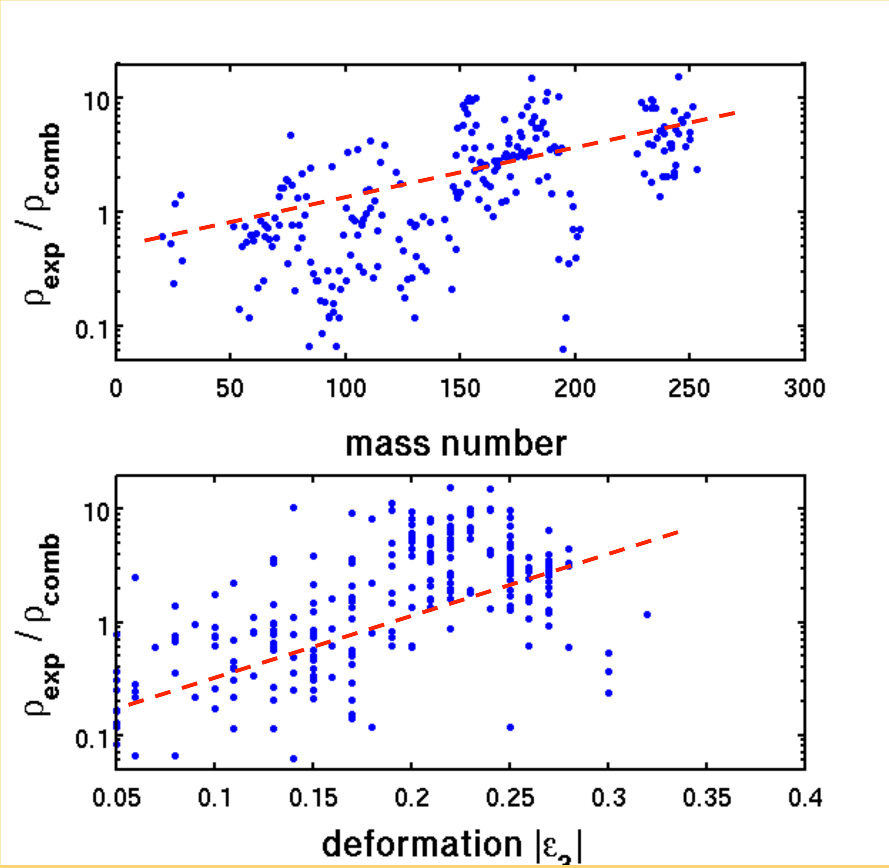
- versus deformation



ρ_{comb}

underestimates
level density
for large
deformations

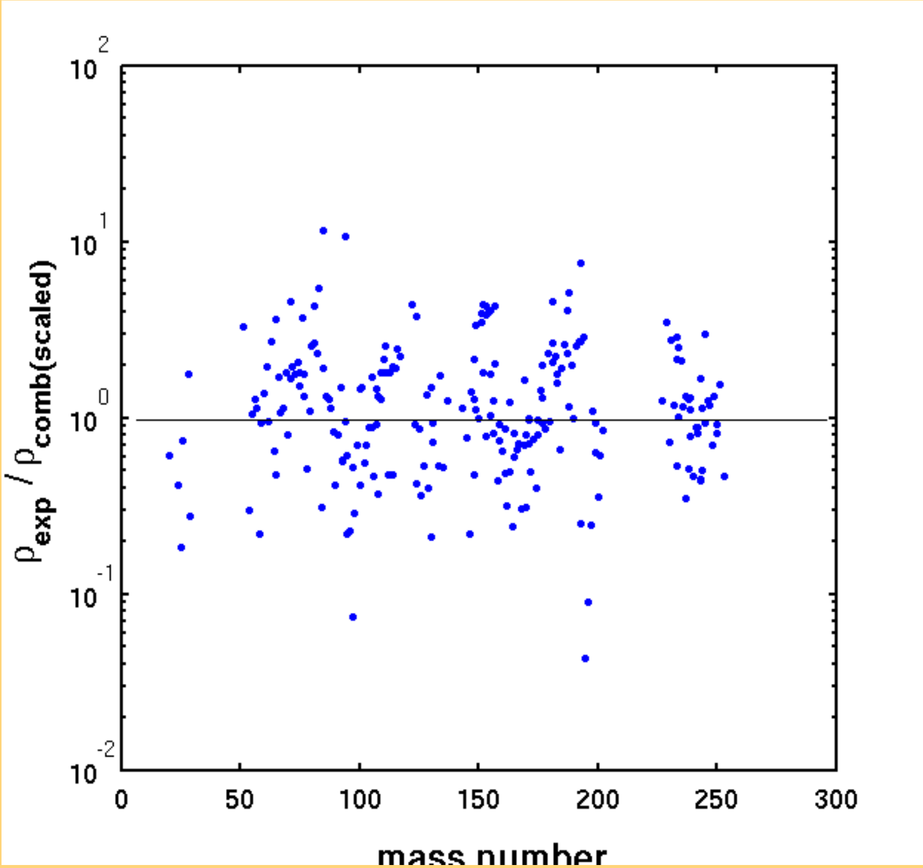
- combined



$\exp(0.01A)$

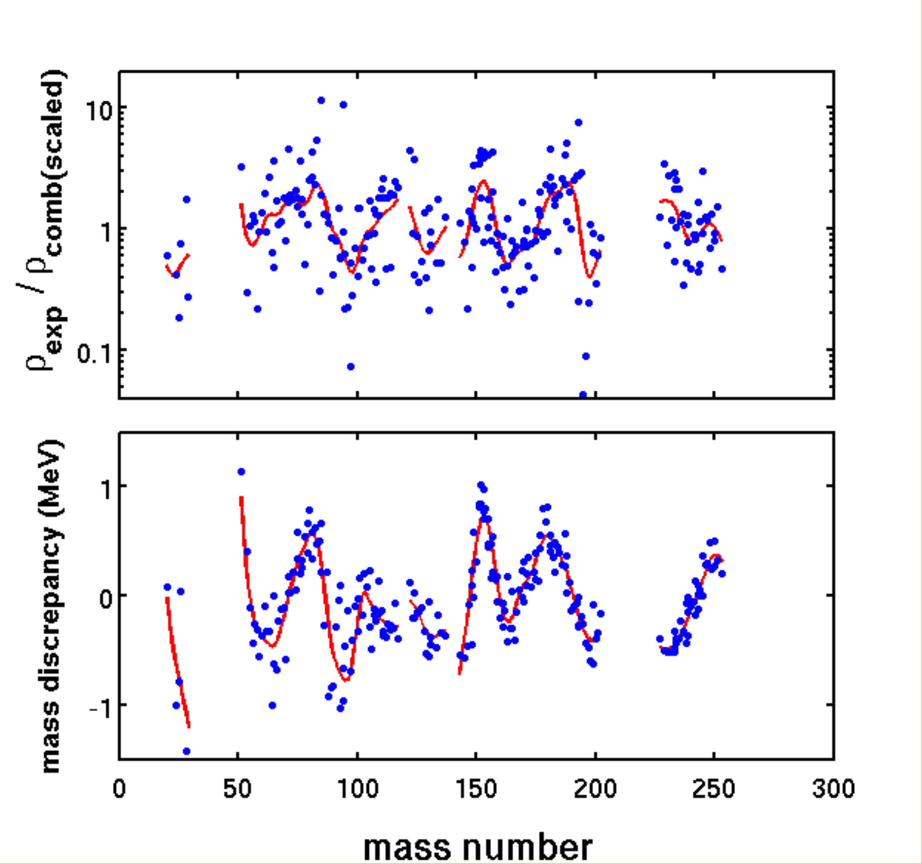
$\exp(10|\varepsilon|)$

Experimental versus scaled combinatorial level density

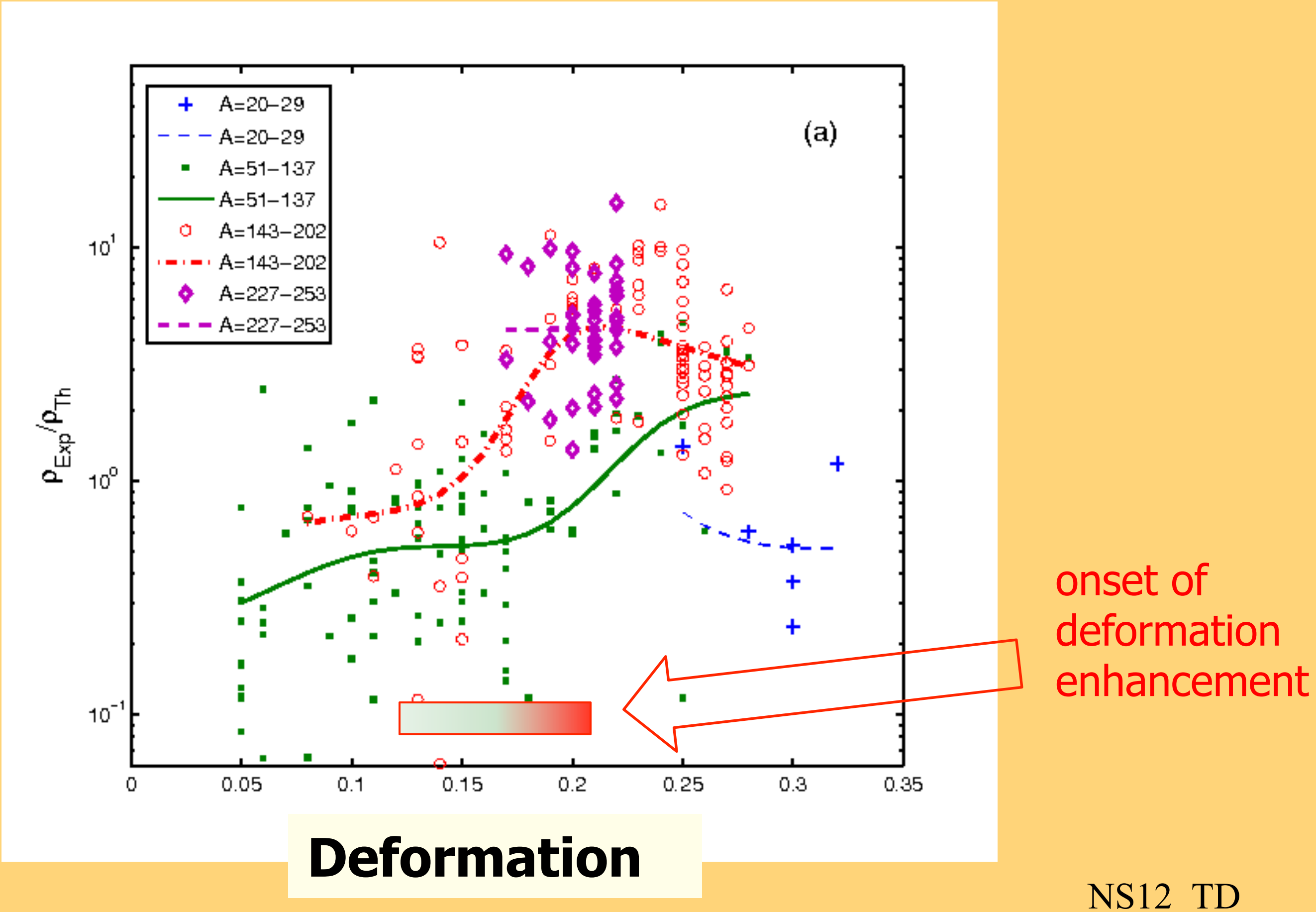


$f_{rms} = 2.3$

- related to discrepancy in mass obtained with same potential



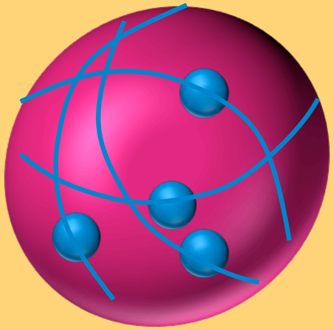
- Experimental relative to combinatorial in different mass regions – inspecting deformation dependence

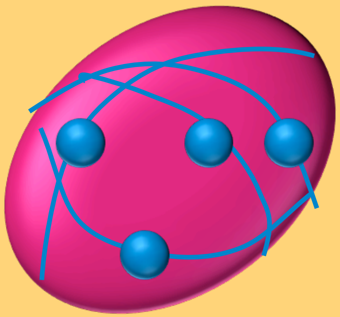


NS12_TD

Angular momentum in many-particle-rotor model

Two, four or six particles in sd shell – coupling and Interacting with deformed core

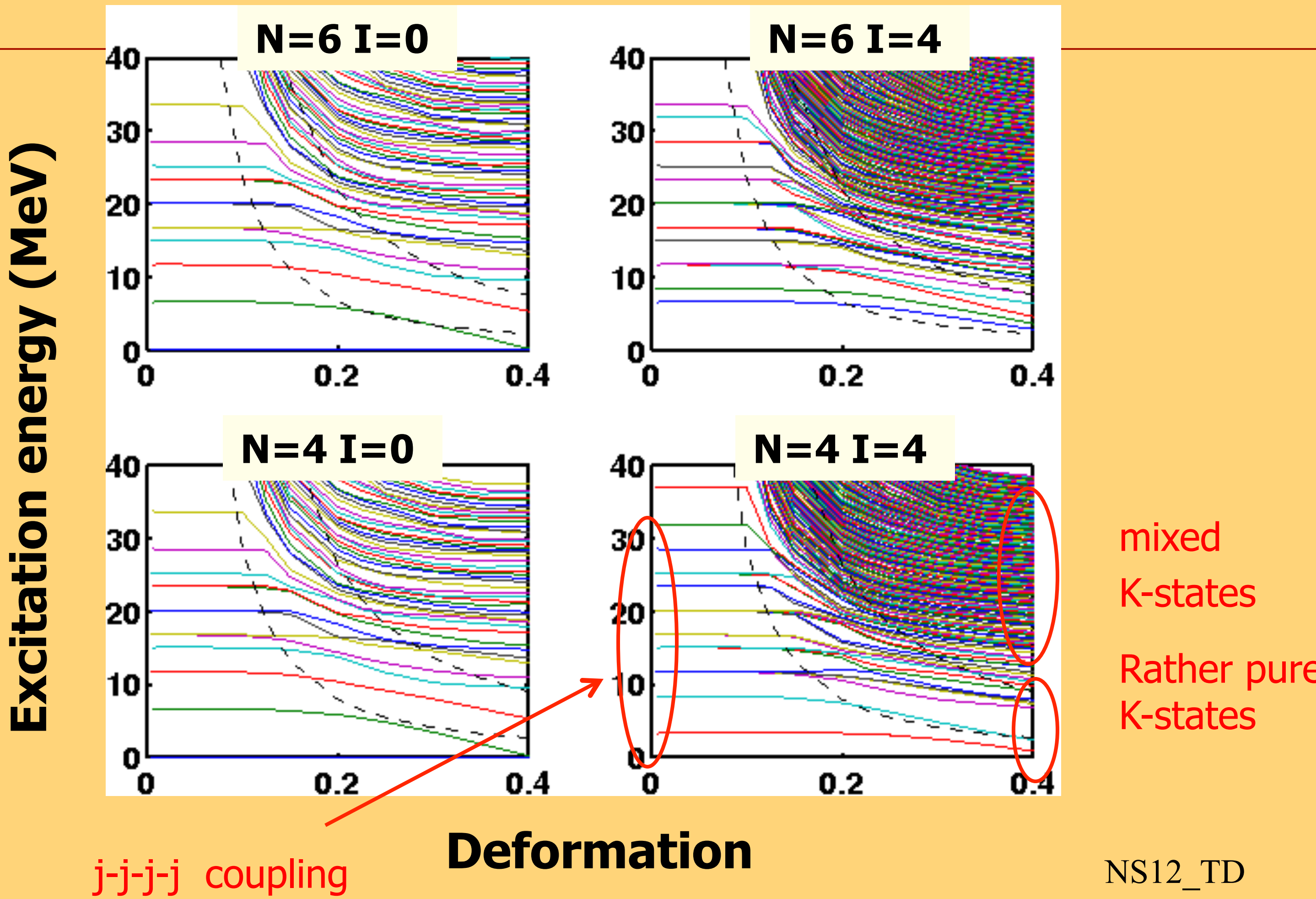




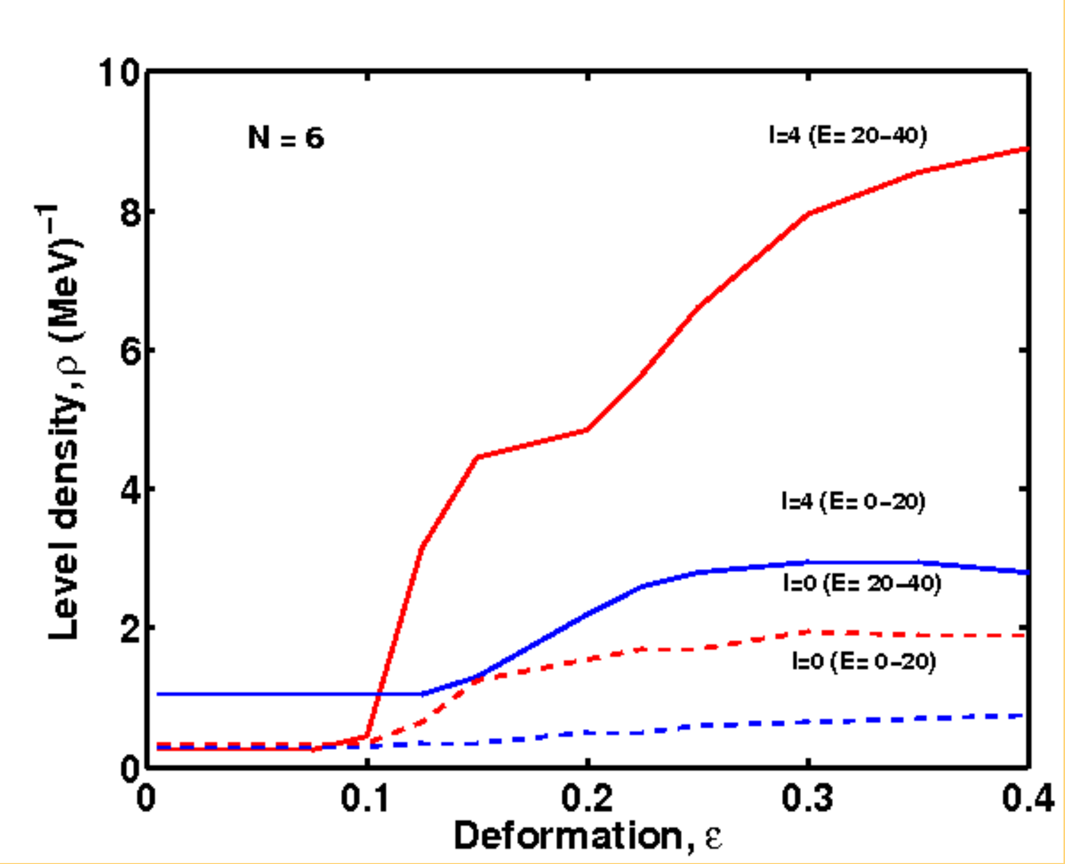
Small deformation:
too costly to rotate -
the nucleons are left on their own to generate the angular momentum vector

Large deformation: the core contributes to the angular momentum vector:
Additional degrees of freedom lead to higher level density

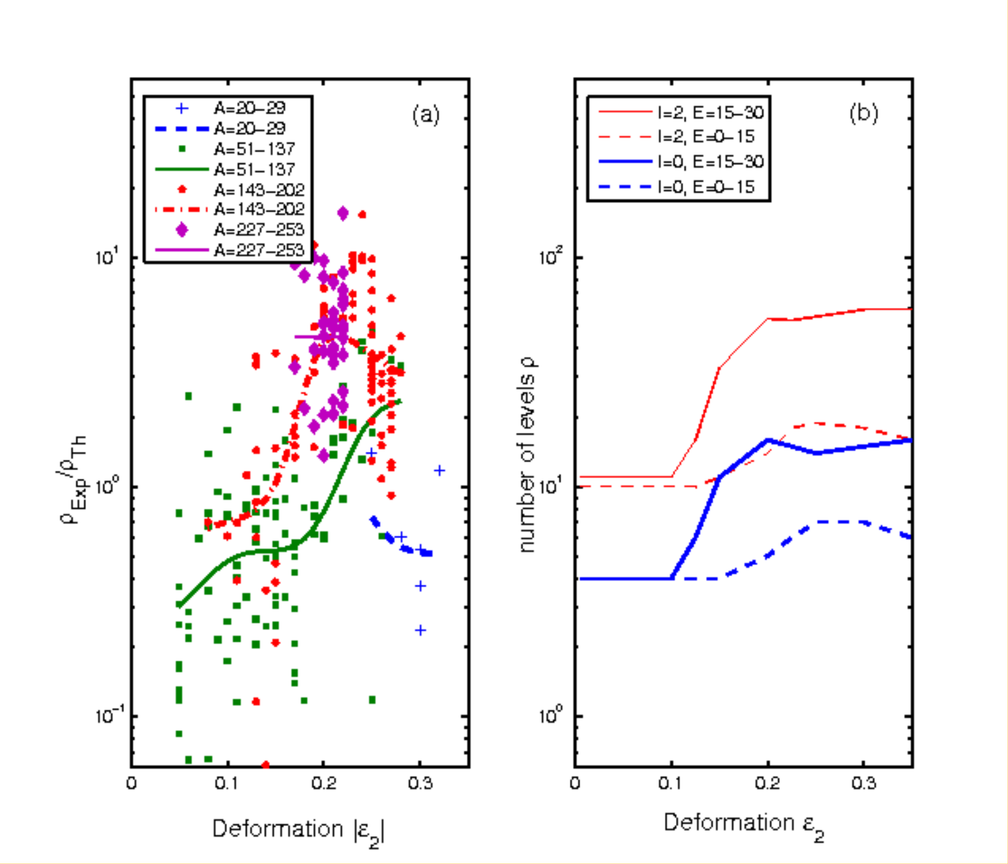
Levels of many-particle rotor model



Level density in many-particle rotor model

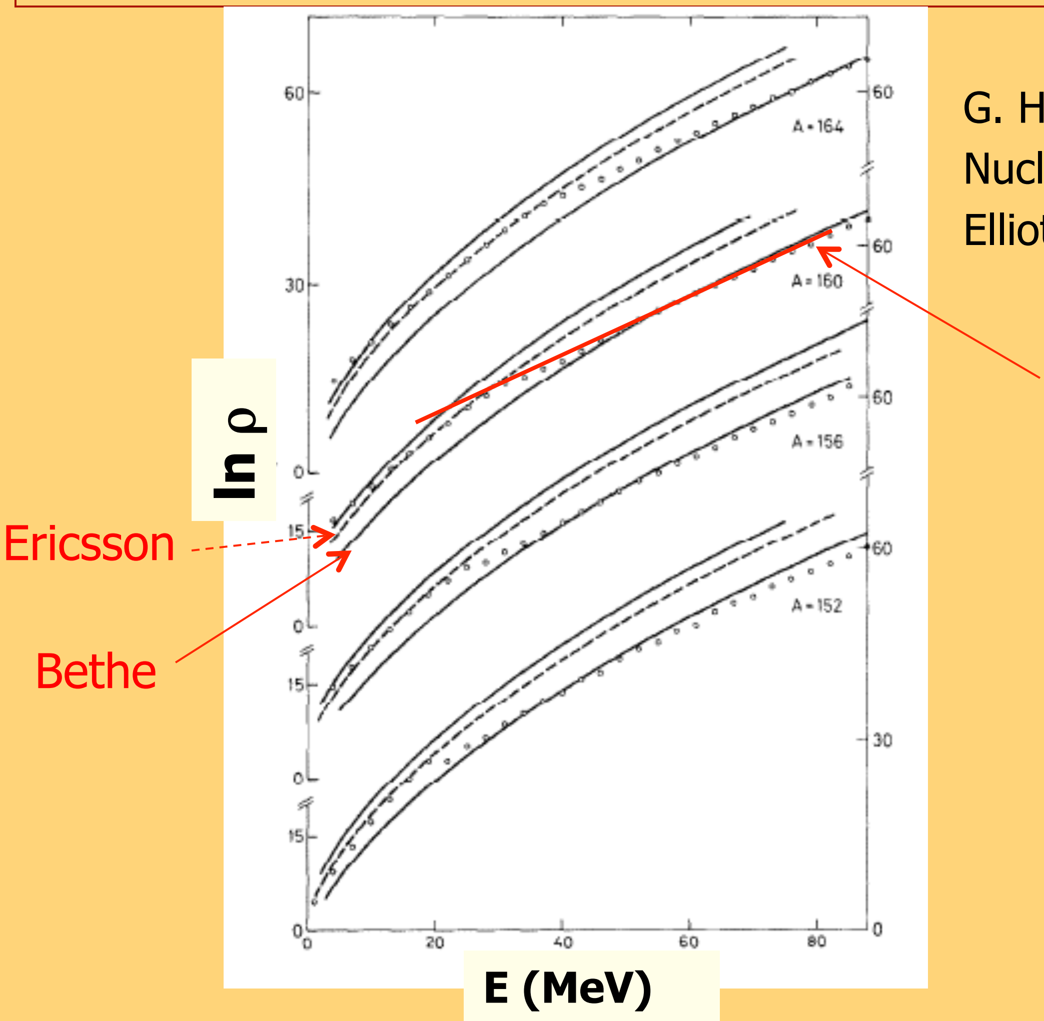


comparison



A=20–29
A=20–29
A=51–137
A=51–137
A=143–202
A=143–202
A=227–253
A=227–253
(a)
ρ_{Exp}/ρ_{Th}
Deformation $|\varepsilon_2|$
I=2, E=15–30
I=2, E=0–15
I=0, E=15–30
I=0, E=0–15
(b)
number of levels ρ
Deformation ε_2

higher up in excitation energy



G. Hansen and A.S. Jensen,
Nucl. Phys. A406 (1983)236:
Elliot model

Conclusion

Combinatorial level density:

Extend Möller-Nix model to thermal properties:

-> mass dependence of level density

- shell structure reproduced
- remaining discrepancy in mass related to remaining discrepancy in level density

-> deformation dependence of level density:

onset of deformation enhancement of level density around

$\varepsilon = 0.15 - 0.2$

Many-particle-rotor level density:

-> precise coupling of angular momentum

-> examine in detail transition from spherical to deformed nuclei