# GFMC Calculations of Electromagnetic Moments and Transitions in $A \leq 9$ Nuclei Including Meson-Exchange Currents $*$

Saori Pastore @ ANL - Argonne, IL - August 2012

$*$ in collaboration with:
Steven Pieper, Rocco Schiavilla, Bob Wiringa,
Luca Girlanda, Michele Viviani, Laura E. Marcucci,
Alejandro Kievsky, José L. Goity

PRC**78**, 064002 (2008) - PRC**80**, 034004 (2009) - PRC**81**, 034005 (2010) - PRL**105**, 232502, (2010) - PRC**84**, 024001 (2011)

- Nuclear Electromagnetic Current Operators from $\chi$EFT
- Fixing Low Energy Constants in $A = 2$–$3$ Systems
- Predictions: EM Observables in $A \leq 9$ Systems
- Summary
- Outlook

# The Basic Model

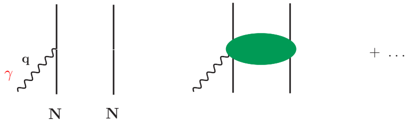
- The nucleus is a system made of A interacting nucleons, its energy is given by

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} \upsilon_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

  where $\upsilon_{ij}$ and $V_{ijk}$ are 2- and 3-nucleon interaction operators

- Current and charge operators describe the interaction of nuclei with external fields. They are expanded as a sum of $1-$, $2-$, ... nucleon operators:

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \ldots \,, \qquad \mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \ldots$$




- EM current operator $\mathbf{j}$ satisfies the current conservation relation (CCR) with the nuclear Hamiltonian, hence V, $\rho$, $\mathbf{j}$ need to be derived consistently

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho]$$

  CCR does not constrain transverse (orthogonal to **q**) currents

# Nuclear $\chi$EFT approach

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett. **B295**, 114 (1992)

- $\chi$EFT exploits the $\chi$ symmetry exhibited by QCD at low energy to restrict the form of the interactions of $\pi$'s with other $\pi$'s, and with $N$'s, $\Delta$'s, ...
- The pion couples by powers of its momentum $Q \rightarrow \mathscr{L}_{eff}$ can be systematically expanded in powers of $Q/\Lambda_\chi$; ($Q \ll \Lambda_\chi \sim 1$ GeV) allowing for a perturbative treatment in terms of Q expansions
- The coefficients of the expansion, Low Energy Constants (LECs) are unknown and need to be fixed by comparison with exp data
- The systematic expansion in $Q$ naturally has the feature

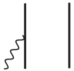
$$\langle \mathscr{O} \rangle_{1-\text{body}} > \langle \mathscr{O} \rangle_{2-\text{body}} > \langle \mathscr{O} \rangle_{3-\text{body}}$$

# Currents from pionful $\chi$EFT

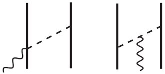
- Rho, Park *et al*. (1996–2009), hybrid studies in A=2–4 Song *et al*. (2009–2011)
- Meissner *et al*. (2001), Kölling *et al*. (2009–2011)
- Phillips (2003), deuteron static properties and f.f.'s

# EM current up to $n = 1$ (or up to N3LO)

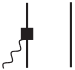
LO : $\mathbf{j}^{(-2)} \sim \mathbf{eQ}^{-2}$



NLO : $\mathbf{j}^{(-1)} \sim \mathbf{eQ}^{-1}$



N$^2$LO : $\mathbf{j}^{(-0)} \sim \mathbf{eQ}^{0}$



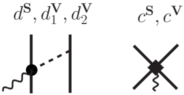
- $n = -2, -1, 0,$ and 1-(loops only): depend on known LECs namely $g_A$, $F_\pi$, and proton and neutron $\mu$
- $n = 0$: $(Q/m_N)^2$ relativistic correction to $\mathbf{j}^{(-2)}$
- unknown LECs enter the $n = 1$ contact and tree-level currents (the latter originates from a $\gamma\pi N$ vertex of order $eQ^2$)

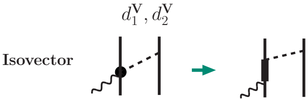
- LECs of contact interactions at $Q^0$ and 'minimal' contact interactions at $Q^2$ fixed from fits to *np* phases shifts: LECs taken from $Q^4$ NN potential of D.R. Entem, R.Machleidt—PRC**68**, 041001 (2003)
- No three-body currents at N3LO

N$^3$LO: $\mathbf{j}^{(1)} \sim \mathbf{eQ}$

unknown LEC′s →

# EM observables at N3LO: fixing LECs - final




Five LECs: $d^S$, $d_1^V$, and $d_2^V$ could be determined by pion photo-production data on the nucleon

$d_2^V$ and $d_1^V$ are known assuming $\Delta$-resonance saturation
$(d_2^V = 4\mu^* h_A/9m(m_\Delta - m)$ and $d_1^V = 4d_2^V)$

Left with 3 LECs: Fixed in the $A = 2-3$ nucleons' sector

* $d^S$ and $c^S$: from EXPT $\mu_d$ and $\mu_S(^3\text{H}/^3\text{He})$
* $c^V$: from EXPT $\mu_V(^3\text{H}/^3\text{He})$ magnetic moment

| $\Lambda$ | NN/NNN | $10 \times d^S/\Lambda^2$ | $c^S/\Lambda^4$ | $d_1^V/\Lambda^2$ | $c^V/\Lambda^4$ |
|---|---|---|---|---|---|
| 600 | AV18/UIX | –2.033 | 5.238 | 4.980 | –1.025 |

Different parameterizations have been studied in the $A = 2, 3$ systems, and tested into the $A = 6, 8$ nuclei (more on this topic on extra slides, if interested)

# Variational Monte Carlo

Minimize expectation value of $H$

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

using trial function

$$|\Psi_V\rangle = \left[ \mathcal{S} \prod_{i<j} (1 + U_{ij} + \sum_{k \neq i,j} U_{ijk}) \right] \left[ \prod_{i<j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

- single-particle $\Phi_A(JMTT_3)$ is fully antisymmetric and translationally invariant
- central pair correlations $f_c(r)$ keep nucleons at favorable pair separation
- pair correlation operators $U_{ij}$ reflect influence of $\upsilon_{ij}$ (AV18)
- triple correlation operator $U_{ijk}$ added when $V_{ijk}$ (IL7) is present

$\Psi_V$ are spin-isospin vectors in $3A$ dimensions with $\sim 2^A \binom{A}{Z}$ components

Lomnitz-Adler, Pandharipande, Smith, NP **A361**, 399 (1981) Wiringa, PRC **43**, 1585 (1991)

# Green's function Monte Carlo

Given a decent trial function $\Psi_V$, we can further improve it by "filtering" out the remaining excited state contamination:

$$\Psi(\tau) = \exp[-(H-E_0)\tau]\Psi_V = \sum_n \exp[-(E_n-E_0)\tau]a_n\psi_n$$

$$\Psi(\tau\to\infty) = a_0\psi_0$$

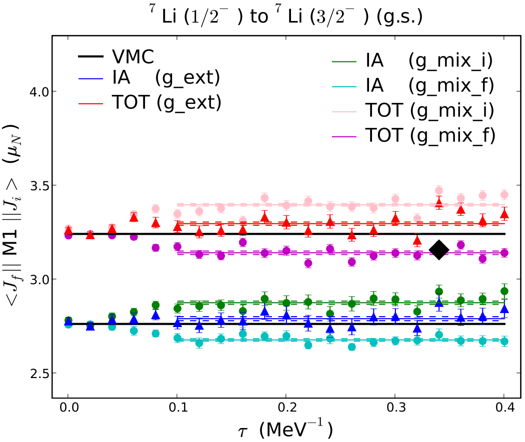
Evaluation of $\Psi(\tau)$ is done stochastically (Monte Carlo method) in small time steps $\Delta\tau$ using a Green's function formulation.
In practice, we evaluate a "mixed" estimates

$$\langle O(\tau)\rangle = \frac{{}_f\langle\Psi(\tau)|O|\Psi(\tau)\rangle_i}{\langle\Psi(\tau)|\Psi(\tau)\rangle} \approx \langle O(\tau)\rangle^i_{\text{Mixed}} + \langle O(\tau)\rangle^f_{\text{Mixed}} - \langle O\rangle_V$$

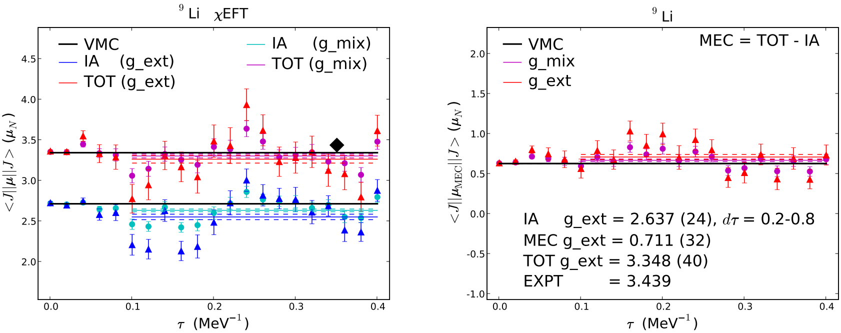
$$\langle O(\tau)\rangle^i_{\text{Mixed}} = \frac{{}_f\langle\Psi_V|O|\Psi(\tau)\rangle_i}{{}_f\langle\Psi_V|\Psi(\tau)\rangle_i}\ ;\ \langle O(\tau)\rangle^f_{\text{Mixed}} = \frac{{}_f\langle\Psi(\tau)|O|\Psi_V\rangle_i}{{}_f\langle\Psi(\tau)|\Psi_V\rangle_i}$$

Pudliner, Pandharipande, Carlson, Pieper, & Wiringa, PRC **56**, 1720 (1997)
Wiringa, Pieper, Carlson, & Pandharipande, PRC **62**, 014001 (2000)
Pieper, Wiringa, & Carlson, PRC **70**, 054325 (2004)

# Examples of GFMC propagation: M1 Transition in $A = 7$



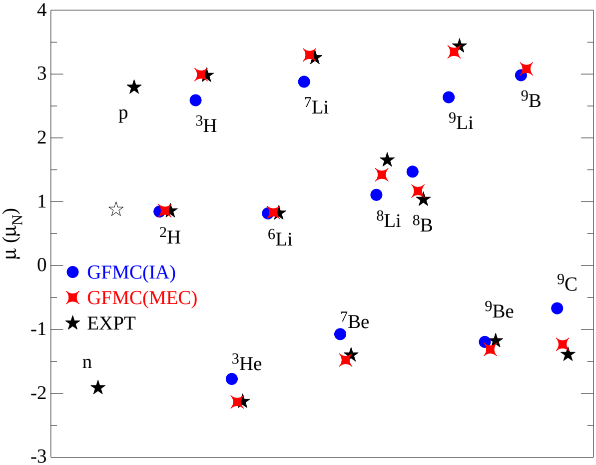
# Examples of GFMC propagation: Magnetic moment in $A = 9$




Reduce noise by increasing the statistic for the IA results

# GFMC calculation of magnetic moments in $A \leq 9$ nuclei: Summary

Predictions for $A > 3$ nuclei




Preliminary results

$$\mu(\text{IA}) = \mu_N \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i (1 - \tau_{i,z})/2]$$

# Magnetic moments in $A \leq 9$ nuclei: SNPA and $\chi$EFT

| | A | s.s. | SNPA | $\chi$EFT* | EXP |
|---|---|---|---|---|---|
| IS | 7 | [43] | 0.840 (18) | 0.911 (11) | 0.929 |
| IV | | [43] | 4.595 (36) | 4.779 (22) | 4.654 |
| IS | 8 | [431] | 1.178 (24) | 1.292 (16) | 1.344 |
| IV | | [431] | -0.146 (24) | -0.142 (16) | -0.310 |
| IS | 9 $[T = 3/2]$ | [432] | 0.927 (45) | 1.058 (29) | 1.024 |
| IV | | [432] | -1.415 (30) | -1.527 (19) | -1.610 |
| IS | 9 $[T = 1/2]$ | [441] | 0.787 (16) | 0.884 (12) | n.a. |
| IV | | [441] | 4.207 (36) | 4.395 (24) | n.a. |

Preliminary results

Overall improvement of isoscalar (IS) component of the magnetic moment

$$\mu = \mu_S + \tau_z \mu_V$$

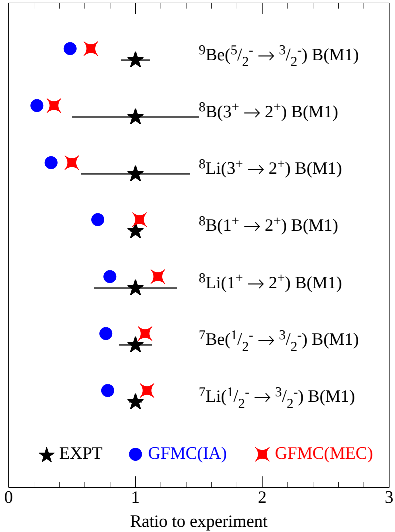
# GFMC calculation of M1 transitions in $A \leq 9$ nuclei: Summary

$$\text{M1(IA)} = \mu_N \sum_i [(L_i + g_p S_i)(1+\tau_{i,z})/2 + g_n S_i (1-\tau_{i,z})/2]$$

Prediction for $^9$Li$(1/2^- \to 3/2^-)$ M1 transition

$\Gamma^{\text{IA}} = 5.93(10)\ (10^{-1}\ \text{eV})$

$\Gamma^{\text{TOT}} = 7.89(25)\ (10^{-1}\ \text{eV})$




Preliminary results

# Summary

- EM current operators have been derived in $\chi$EFT up to $n = 1$
- Predictions from hybrid calculations of magnetic moment and M1 transitions in $A \leq 9$ nuclei are in good agreement with experimental data: Corrections of order $>$ LO are important to bring theory in agreement with experimental data

## Outlook: electroweak properties of light nuclei

* EM structure of light nuclei
    - Extend hybrid calculations to different combinations of 2N and 3N potentials to study charge radii, charge and magnetic form factors of $A \leq 10$ systems (on going project)

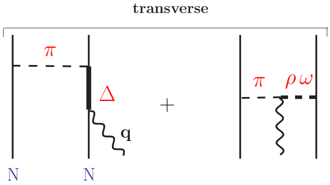
* Weak structure of light nuclei
    - Extend hybrid calculations to weak properties of light nuclei

# EXTRA SLIDES

# Currents from nuclear interactions ∗- Marcucci *et al.* PRC**72**, 014001 (2005)

- Current operator **j** constructed so as to satisfy the continuity equation with a realistic Hamiltonian
- Short- and intermediate-behavior of the EM operators inferred from the nuclear two- and three-body potentials

$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V)$$

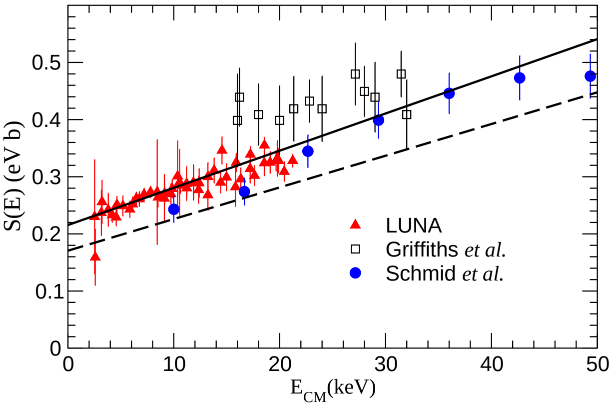



∗ also referred to as Standard Nuclear Physics Approach (SNPA) currents

# Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)

Satisfactory description of a variety of nuclear EM properties [see Marcucci *et al.* (2005) and (2008)]

$^2$H(p,$\gamma$)$^3$He capture




- Isoscalar magnetic moments are a few % off (10% in $A$=7 nuclei)

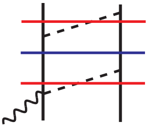
# Transition amplitude in time-ordered perturbation theory

$$T_{\rm fi} = \langle f \mid T \mid i \rangle = \langle f \mid H_1 \sum_{n=1}^{\infty} \left( \frac{1}{E_i - H_0 + i\eta} H_1 \right)^{n-1} \mid i \rangle$$

$$= \langle f \mid H_1 \mid i \rangle + \sum_{|I\rangle} \langle f \mid H_1 \mid I \rangle \frac{1}{E_i - E_I} \langle I \mid H_1 \mid i \rangle + \dots$$

- A contribution with N interaction vertices and L loops scales as

$$\underbrace{e \left( \prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2} \right)}_{H_1\,\text{scaling}} \times \underbrace{Q^{-(N-N_K-1)}\, Q^{-2N_K}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integration}}$$



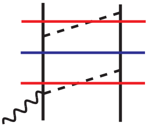
$\alpha_i$ = number of derivatives in $H_1$ and $\beta_i$ = number of $\pi$'s at each vertex

$N_K$ = number of pure nucleonic intermediate states

- Due to the chiral expansion, the transition amplitude $T_{\rm fi}$ can be expanded as

$$T_{\rm fi} = T^{\rm LO} + T^{\rm NLO} + T^{\rm N2LO} + \dots \quad \text{and} \quad T^{\rm NnLO} \sim (Q/\Lambda_\chi)^n T^{\rm LO}$$

# Power counting



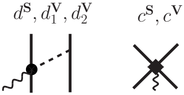
- $N_K$ energy denominators scale as $Q^{-2}$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N}|I\rangle \sim Q^{-2}|I\rangle$$

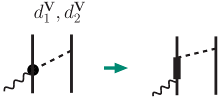
- $(N - N_K - 1)$ energy denominators scale $Q^{-1}$ in the static limit; they can be further expanded in powers of $(E_i - E_N)/\omega_\pi \sim Q$

$$\frac{1}{E_i - E_I}|I\rangle = \frac{1}{E_i - E_N - \omega_\pi}|I\rangle \sim -\Big[\underbrace{\frac{1}{\omega_\pi}}_{Q^{-1}} + \underbrace{\frac{E_i - E_N}{\omega_\pi^2}}_{Q^0} + \underbrace{\frac{(E_i - E_N)^2}{\omega_\pi^3}}_{Q^1} + \ldots\Big]|I\rangle$$

- Terms accounted into the Lippmann-Schwinger Eq. are subtracted from the reducible amplitude
- EM operators depend on the off-the-energy shell prescription adopted for the non-static OPE and TPE potentials
- Ultimately, the EM operators are unitarily equivalent: Description of physical systems is not affected by this ambiguity




Five LECs: $d^S$, $d_1^V$, and $d_2^V$ could be determined by pion photo-production data on the nucleon



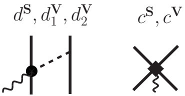

$d_2^V$ and $d_1^V$ are known assuming $\Delta$-resonance saturation ($d_2^V/d_1^V = 1/4$ )

Left with 5 LECs: Fixed in the $A = 2-3$ nucleons' sector

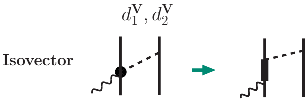
- Isoscalar sector:
  - * $d^S$ and $c^S$ from EXPT $\mu_d$ and $\mu_S(^3\text{H}/^3\text{He})$

| $\Lambda$ | NN/NNN | $10 \times d^S/\Lambda^2$ | $c^S/\Lambda^4$ |
|---|---|---|---|
| 600 | AV18/UIX | –2.033 | 5.238 |

# EM observables at N3LO: fixing LECs p.2/3




Five LECs: $d^S$, $d_1^V$, and $d_2^V$ could be determined by pion photo-production data on the nucleon




$d_2^V$ and $d_1^V$ are known assuming $\Delta$-resonance saturation ($d_2^V/d_1^V = 1/4$ )

Left with 4 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isovector sector:
  - * I = $c^V$ and $d_1^V$ from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. and EXPT $npd\gamma$ xsec.

    or
  - * II = $c^V$ from EXPT $npd\gamma$ xsec. and $d_1^V$ from $\Delta$-saturation

    or
  - * III = $c^V$ from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. and $d_1^V$ from $\Delta$-saturation*

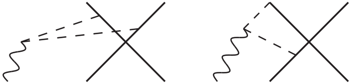
| $\Lambda$ | NN/NNN | Current | $d_1^V/\Lambda^2$ | $c^V/\Lambda^4$ |
|---|---|---|---|---|
| 600 | AV18/UIX | I | 75.0 | 257.5 |
| | | II | 4.98 | -11.57 |
| | | III | 4.98 | -1.025 |

* $d_1^V = 4\mu^* h_A / 9m(m_\Delta - m)$

Table: m.m.'s of $^3$H, $^7$Li, and $^8$Li, with currents I or III and $\Lambda$=600 MeV from VMC

| Nucleus | Current | IA | NLO | N2LO | N3LO | LECs(tree) | LECs(ct) | SUM |
|---|---|---|---|---|---|---|---|---|
| $^3$H | I | 2.590 | 0.253 | -0.033 | 0.091 | 1.612 | -1.555 | 2.958 |
| | III | | | | | 0.102 | -0.011 | 2.992 |
| $^7$Li | I | 2.899 | 0.254 | -0.064 | 0.081 | 1.718 | -1.855 | 3.033 |
| | III | | | | | 0.109 | -0.011 | 3.268 |
| $^8$Li | I | 1.258 | 0.223 | -0.039 | 0.088 | 1.084 | -1.541 | 1.073 |
| | III | | | | | 0.066 | -0.015 | 1.581 |

# 2009 EM current vs 2011 EM currents p. 1/2




- Non-static corrections entering single-nucleon operators accounted into the derivation of current i)

$$
\begin{aligned}
i)\,\text{OLD} &= i\frac{e\,g_A^2}{F_\pi^2}\,\tau_{1,z}\int\frac{\mathbf{q}_1-\mathbf{q}_2}{\omega_1^3\,\omega_2^3}\,\frac{\omega_1^2+\omega_1\,\omega_2+\omega_2^2}{\omega_1+\omega_2}\Big[C_S\,\boldsymbol{\sigma}_1\cdot(\mathbf{q}_1\times\mathbf{q}_2)\\
&- C_T\,\boldsymbol{\sigma}_2\cdot(\mathbf{q}_1\times\mathbf{q}_2)\Big]+1\rightleftharpoons 2\\
i)\,\text{NEW} &= 2\,i\,\frac{e\,g_A^2\,C_T}{F_\pi^2}\,\tau_{1z}\int_{\mathbf{q}_1,\mathbf{q}_2}\frac{\omega_1^2+\omega_1\,\omega_2+\omega_2^2}{\omega_1^3\omega_2^3(\omega_1+\omega_2)}\,(\mathbf{q}_1-\mathbf{q}_2)\,\boldsymbol{\sigma}_2\cdot\mathbf{q}_2\times\mathbf{q}_1+1\rightleftharpoons 2
\end{aligned}
$$

- i) NEW in agreement with Kölling 2009/2011* but for a factor of 2, which has no impact because (i + g) = 0

* PRC**80**, 045502 (2009)/ PRC **84**, 054008 (2011)


- A different derivation in Kölling 2009/2011* leads to an additional term $\sim (\boldsymbol{\sigma}_i \times \mathbf{q}) \times \mathbf{q}$ in the N3LO current at tree level, which however does not contribute to the magnetic moment
- The N3LO contact current of Pastore 2009 is in agreement with that of Kölling 2011 after Fierz-reordering, apart from differences in the term $\propto C_5$:

$$\mathbf{j}_{\rm ct}^{\rm N3LO} = -\frac{i\,C_5}{4}\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times (e_1\,\mathbf{k}_1 + e_2\,\mathbf{k}_2)$$

* PRC**80**, 045502 (2009)/ PRC **84**, 054008 (2011)

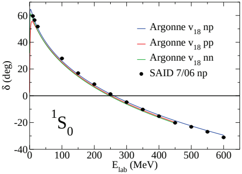
## NUCLEAR HAMILTONIAN


$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

$K_i$: Non-relativistic kinetic energy, $m_n$-$m_p$ effects included

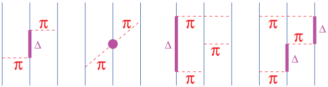
Argonne v18: $v_{ij} = v^{\gamma}_{ij} + v^{\pi}_{ij} + v^{I}_{ij} + v^{S}_{ij} = \sum v_p(r_{ij}) O^p_{ij}$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with $\chi^2$/d.o.f.=1.1




Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V^{2\pi}_{ijk} + V^{3\pi}_{ijk} + V^{R}_{ijk}$



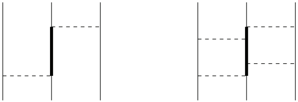

- Urbana has standard $2\pi$ $P$-wave + short-range repulsion for matter saturation
- Illinois adds $2\pi$ $S$-wave + $3\pi$ rings to provide extra $T$=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

courtesy of R.B.Wiringa

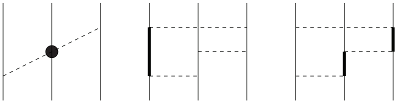
## THREE-NUCLEON POTENTIALS

Urbana $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{R}$



Carlson, Pandharipande, & Wiringa, NP **A401**, 59 (1983)

Illinois $V_{ijk} = V_{ijk}^{2\pi P} + V_{ijk}^{2\pi S} + V_{ijk}^{3\pi\Delta R} + V_{ijk}^{R}$

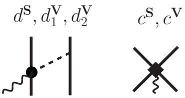


Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)

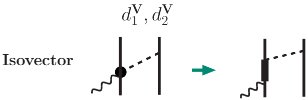
Illinois-7 has 4 strength parameters fit to 23 energy levels in $A \leq 10$ nuclei.
In light nuclei we find (thanks to large cancellation between $\langle K \rangle$ & $\langle v_{ij} \rangle$):

$$\langle V_{ijk} \rangle \sim (0.02 \text{ to } 0.07)\ \langle v_{ij} \rangle \sim (0.15 \text{ to } 0.5)\ \langle H \rangle$$

We expect $\langle \mathcal{V}_{ijkl} \rangle \sim 0.05\ \langle V_{ijk} \rangle \sim (0.01 \text{ to } 0.03)\ \langle H \rangle \sim 1$ MeV in $^{12}$C .




Five LECs: $d^S$, $d_1^V$, and $d_2^V$ could be determined by pion photo-production data on the nucleon



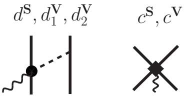

$d_2^V$ and $d_1^V$ are known assuming $\Delta$-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 5 LECs: Fixed in the $A = 2-3$ nucleons' sector

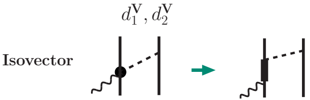
- Isoscalar sector:
  - $d^S$ and $c^S$ from EXPT $\mu_d$ and $\mu_S(^3\text{H}/^3\text{He})$

| $\Lambda$ | NN/NNN | $10 \times d^S/\Lambda^2$ | $c^S/\Lambda^4$ |
|---|---|---|---|
| 600 | AV18/UIX | –2.033 | 5.238 |

# EM observables at N3LO: fixing LECs p.2/3




Five LECs: $d^S$, $d_1^V$, and $d_2^V$ could be determined by pion photo-production data on the nucleon




$d_2^V$ and $d_1^V$ are known assuming $\Delta$-resonance saturation ($d_2^V/d_1^V = 1/4$)

Left with 4 LECs: Fixed in the $A = 2-3$ nucleons' sector

- Isovector sector:
  - * I = $c^V$ and $d_1^V$ from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. and EXPT $npd\gamma$ xsec.

    or
  - * II = $c^V$ from EXPT $npd\gamma$ xsec. and $d_1^V$ from $\Delta$-saturation

    or
  - * III = $c^V$ from EXPT $\mu_V(^3\text{H}/^3\text{He})$ m.m. and $d_1^V$ from $\Delta$-saturation*

| $\Lambda$ | NN/NNN | Current | $d_1^V/\Lambda^2$ | $c^V/\Lambda^4$ |
|---|---|---|---|---|
| 600 | AV18/UIX | I | 75.0 | 257.5 |
| | | II | 4.98 | -11.57 |
| | | III | 4.98 | -1.025 |

* $d_1^V = 4\mu^* h_A/9m(m_\Delta - m)$

Table: m.m.'s of $^3$H, $^7$Li, and $^8$Li, with currents I or III and $\Lambda$=600 MeV from VMC

| Nucleus | Current | IA | NLO | N2LO | N3LO | LECs(tree) | LECs(ct) | SUM |
|---|---|---|---|---|---|---|---|---|
| $^3$H | I | 2.590 | 0.253 | -0.033 | 0.091 | 1.612 | -1.555 | 2.958 |
| | III | | | | | 0.102 | -0.011 | 2.992 |
| $^7$Li | I | 2.899 | 0.254 | -0.064 | 0.081 | 1.718 | -1.855 | 3.033 |
| | III | | | | | 0.109 | -0.011 | 3.268 |
| $^8$Li | I | 1.258 | 0.223 | -0.039 | 0.088 | 1.084 | -1.541 | 1.073 |
| | III | | | | | 0.066 | -0.015 | 1.581 |

# Magnetic moment at N$^3$LO

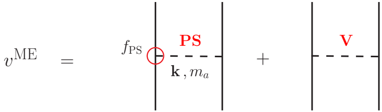
- Magnetic moment operator due to two-body current density $\mathbf{J}(\mathbf{x})$

$$\boldsymbol{\mu}(\mathbf{R},\mathbf{r}) = \frac{1}{2}\left[\mathbf{R}\times\int d\mathbf{x}\,\mathbf{J}(\mathbf{x}) + \int d\mathbf{x}\,(\mathbf{x}-\mathbf{R})\times\mathbf{J}(\mathbf{x})\right]$$

- Sachs' and translationally invariant magnetic moments

$$\begin{aligned}\boldsymbol{\mu}_{\rm Sachs}(\mathbf{R},\mathbf{r}) &= -i\,\frac{\mathbf{R}}{2}\times\int d\mathbf{x}\,\mathbf{x}\,[\rho(\mathbf{x})\,,\,\upsilon_{12}] \\ \boldsymbol{\mu}_{\rm T}(\mathbf{r}) &= -\frac{i}{2}\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}\,\nabla_q\times\mathbf{j}(\mathbf{q},\mathbf{k})\Big|_{\mathbf{q}=0}\end{aligned}$$

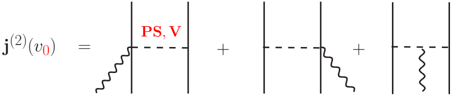
# Currents from nuclear interactions - Marcucci *et al.* PRC**72**, 014001 (2005)



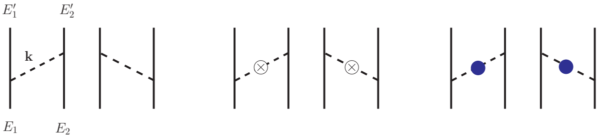

- Exploiting the meson exchange (ME) mechanism, one assumes that the static part $\upsilon_0$ of $\upsilon$ is due to pseudoscalar (PS) and vector (V) exchanges
- $\upsilon^{ME}$ is expressed in terms of 'effective propagators' $\upsilon_{PS}$, $\upsilon_V$, $\upsilon_{VS}$, fixed such to reproduce $\upsilon_0$, for example

$$\upsilon_{PS} = [\upsilon^{\sigma\tau}(k) - 2\upsilon^{t\tau}(k)]/3$$

  with $\upsilon^{\sigma\tau}$ and $\upsilon^{t\tau}$ components of $\upsilon_0$
- The current operator is obtained by taking the non relativistic reduction of the ME Feynman amplitudes and replacing the bare propagators with the 'effective' ones



# OPEP beyond the static limit




$v_\pi^{(0)} \sim Q^0$ $\qquad$ $v_\pi^{(1)} \sim Q^1$ $\qquad$ $v_\pi^{(2)} \sim Q^2$

On-the-energy-shell, non-static OPEP at N2LO ($Q^2$) can be equivalently written as

$$
\begin{aligned}
\upsilon_\pi^{(2)}(\nu = 0) &= \upsilon_\pi^{(0)}(\mathbf{k})\, \frac{(E_1' - E_1)^2 + (E_2' - E_2)^2}{2\,\omega_k^2} \\
\upsilon_\pi^{(2)}(\nu = 1) &= -\upsilon_\pi^{(0)}(\mathbf{k})\, \frac{(E_1' - E_1)\,(E_2' - E_2)}{\omega_k^2} \\
\upsilon_\pi^{(0)}(\mathbf{k}) &= -\frac{g_A^2}{F_\pi^2}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{k}\, \boldsymbol{\sigma}_2 \cdot \mathbf{k}}{\omega_k^2}
\end{aligned}
$$

$\upsilon_\pi^{(2)}(\nu)$ corrections are different off-the-energy-shell $(E_1 + E_2 \neq E_1' + E_2')$

- TPE contributions are affected by the choice made for the parameter $\nu$

# From amplitudes to potentials

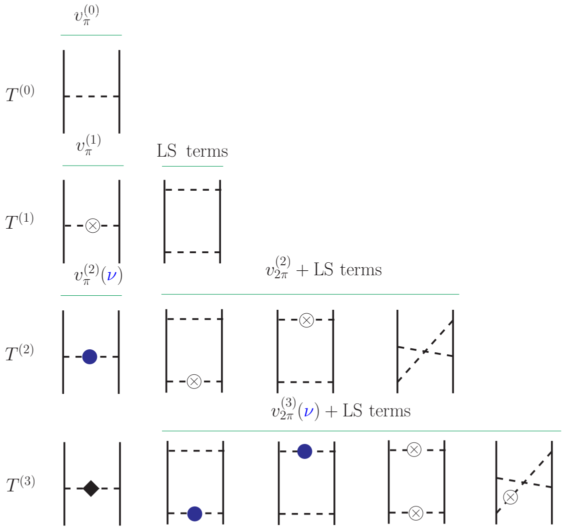
The two-nucleon potential $\upsilon = \upsilon^{(0)} + \upsilon^{(1)} + \upsilon^{(2)} + \dots$ (with $\upsilon^{(n)} \sim Q^n$) is iterated into the Lippmann-Schwinger (LS) equation *i.e.*

$$\upsilon + \upsilon\, G_0\, \upsilon + \upsilon\, G_0\, \upsilon\, G_0\, \upsilon + \dots\,, \qquad\qquad G_0 = 1/(E_i - E_I + i\,\eta)$$

$\upsilon^{(n)}$ is obtained subtracting from the transition amplitude $T_{\rm fi}^{(n)}$ terms already accounted for into the LS equation

$$\begin{aligned}
\upsilon^{(0)} &= T^{(0)}\,, \\
\upsilon^{(1)} &= T^{(1)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right]\,, \\
\upsilon^{(2)} &= T^{(2)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(1)}\right]\,, \\
\upsilon^{(3)}(\nu) &= T^{(3)} - \left[\upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)}\right] - \left[\upsilon^{(1)}\, G_0\, \upsilon^{(0)}\, G_0\, \upsilon^{(0)} + \text{permutations}\right] \\
&\quad \underbrace{- \left[\upsilon^{(1)}\, G_0\, \upsilon^{(1)}\right] - \left[\upsilon^{(2)}(\nu)\, G_0\, \upsilon^{(0)} + \upsilon^{(0)}\, G_0\, \upsilon^{(2)}(\nu)\right]}_{\text{LS terms}}
\end{aligned}$$

# From amplitudes to potentials: an example with OPE and TPE only




- To each $\upsilon_\pi^{(2)}(\nu)$ corresponds a $\upsilon_{2\pi}^{(3)}(\nu)$

# Unitary equivalence of $\upsilon_\pi^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$

- Different off-the-energy-shell parameterizations lead to unitarily equivalent two-nucleon Hamiltonians

$$H(\nu) = t^{(-1)} + \upsilon_\pi^{(0)} + \upsilon_{2\pi}^{(2)} + \upsilon_\pi^{(2)}(\nu) + \upsilon_{2\pi}^{(3)}(\nu)$$

$t^{(-1)}$ is the kinetic energy, $\upsilon_\pi^{(0)}$ and $\upsilon_{2\pi}^{(2)}$ are the static OPEP and TPEP

- The Hamiltonians are related to each other via

$$H(\nu) = \mathrm{e}^{-iU(\nu)} H(\nu=0)\, \mathrm{e}^{+iU(\nu)} \,, \qquad i\,U(\nu) \simeq i\,U^{(0)}(\nu) + i\,U^{(1)}(\nu)$$

from which it follows

$$H(\nu) = H(\nu=0) + \left[t^{(-1)} + \upsilon_\pi^{(0)} \,, i\,U^{(0)}(\nu)\right] + \left[t^{(-1)}, i\,U^{(1)}(\nu)\right]$$

- Predictions for physical observables are unaffected by off-the-energy-shell effects

## From amplitudes to EM charge and current operators

- In presence of EM interaction the transition amplitude $T_\gamma$ is expanded as

$$T_\gamma = T_\gamma^{(-3)} + T_\gamma^{(-2)} + T_\gamma^{(-1)} + \dots, \qquad T_\gamma^{(n)} \sim e\,Q^n$$

and the charge and current operators are related to $T_\gamma^{(n)}$ via

$$\upsilon_\gamma^{(n)} = A^0 \rho^{(n)} - \mathbf{A}\cdot\mathbf{j}^{(n)} = T_\gamma^{(n)} - \text{ LS terms}$$

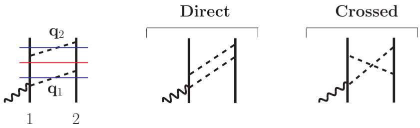
that is

$$\begin{aligned}
\upsilon_\gamma^{(-3)} &= T_\gamma^{(-3)} \,, \\
\upsilon_\gamma^{(-2)} &= T_\gamma^{(-2)} - \left[\upsilon_\gamma^{(-3)} G_0\, \upsilon^{(0)} + \upsilon^{(0)} G_0\, \upsilon_\gamma^{(-3)}\right] \,, \\
\upsilon_\gamma^{(-1)} &= T_\gamma^{(-1)} \underbrace{- \left[\upsilon_\gamma^{(-3)} G_0\, \upsilon^{(0)} G_0\, \upsilon^{(0)} + \text{permutations}\right] - \left[\upsilon_\gamma^{(-2)} G_0\, \upsilon^{(0)} + \upsilon^{(0)} G_0\, \upsilon_\gamma^{(-2)}\right]}_{\text{LS terms}}
\end{aligned}$$

$\dots\dots$

# Technical issue II - Recoil corrections at N$^3$LO

$\mathbf{j}^{\mathrm{N^3LO}} \quad =$




- Reducible contributions

$$\mathbf{j}_{\mathrm{red}} \sim \int \upsilon^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1) - \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

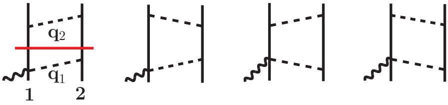
- Irreducible contributions

$$\mathbf{j}_{\mathrm{irr}} = \int 2\,\frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1) - \int 2\,\frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2\,(\omega_1+\omega_2)}\,[V_{\pi NN}(2,\mathbf{q}_2), V_{\pi NN}(2,\mathbf{q}_1)]_{-}\,V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

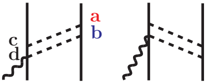
- Observed partial cancellations at N$^3$LO between recoil corrections to reducible diagrams and irreducible contributions

# The box diagram: an example at N$^3$LO

**Reducible**




**Irreducible direct**



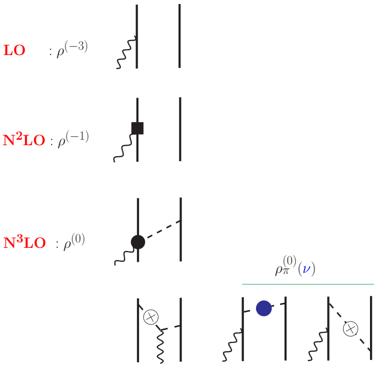

**Irreducible crossed**


$$\text{direct} = f_d(\omega_1,\omega_2)\, V_a\, V_b\, V_c\, V_d$$

$$\text{crossed} = f_c(\omega_1,\omega_2)\, V_b\, V_a\, V_c\, V_d \qquad V_b V_a = V_a V_b - [V_a, V_b]_-$$

$$\begin{aligned}\text{irreducible} &= [f_d(\omega_1,\omega_2) + f_c(\omega_1,\omega_2)] V_a\, V_b\, V_c\, V_d \\ &- f_c(\omega_1,\omega_2)[V_a, V_b]_-\, V_c\, V_d\end{aligned}$$

# EM charge up to $n = 0$ (or up to N3LO)




- $n = -3$

  $\rho^{(-3)}(\mathbf{q}) = e\,(2\pi)^3\delta(\mathbf{p}_1 + \mathbf{q} - \mathbf{p}_1')\,(1 + \tau_{1,z})/2 + 1 \rightleftharpoons 2$

- $n = -1$:

  $(Q/m_N)^2$ relativistic correction to $\rho^{(-3)}$

- $n = 0$:

  i) 'static' tree-level current (originates from a $\gamma\pi N$ vertex of order $e\,Q$)

  ii) 'non-static' OPE charge operators, $\rho_\pi^{(0)}(\nu)$ depends on $\upsilon_\pi^{(2)}(\nu)$

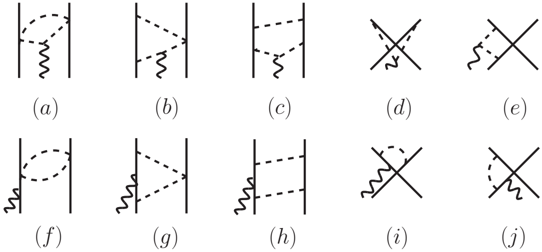
- $\rho_\pi^{(0)}(\nu)$'s are unitarily equivalent

$$\rho_\pi^{(0)}(\nu) \quad = \quad \rho_\pi^{(0)}(\nu = 0) + \left[\rho^{(-3)}\,,\, i\,U^{(0)}(\nu)\right]$$

- No unknown LECs up to this order ($g_A$, $F_\pi$)

# EM charge @ $n = 1$ (or N4LO) 1.

**N⁴LO** : $\rho^{(1)}$

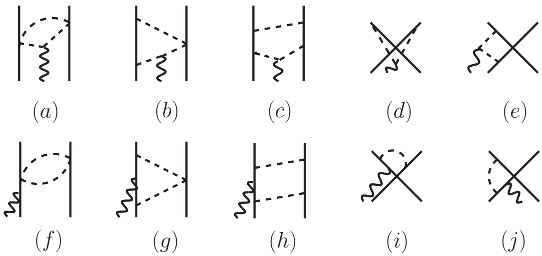



- (a), (f), (d), and (i) vanish
- Divergencies associated with (b) + (g), (c) + (h), and (e) + (j) cancel out—as they must since there are no counter-terms at N4LO
- $\rho_{\rm h}^{(1)}(\nu)$ depends on the parametrization adopted for $\upsilon_{\pi}^{(2)}(\nu)$ and $\upsilon_{2\pi}^{(3)}(\nu)$
- $\rho_{\rm h}^{(1)}(\nu)$'s are unitarily equivalent

$$\rho_{\rm h}^{(1)}(\nu) \quad = \quad \rho_{\rm h}^{(1)}(\nu = 0) + \left[\rho^{(-3)}, i\,U^{(1)}(\nu)\right]$$

# EM charge @ $n=1$ (or N4LO) 2.

**N$^4$LO** : $\rho^{(1)}$




- Charge operators ($\nu$-dependent included) up to $n=1$ satisfy the condition

$$\rho^{(n>-3)}(\mathbf{q}=0)=0$$

which follows from charge conservation

$$\rho(\mathbf{q}=0)=\int \mathrm{d}\mathbf{x}\,\rho(\mathbf{x})=e\frac{(1+\tau_{1,z})}{2}+1\rightleftharpoons 2=\rho^{(-3)}(\mathbf{q}=0)$$

- $\rho^{(1)}$ does not depend on unknown LECs and it is purely isovector