The No Core Shell Model in an Effective Field Theory Framework

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MOTIVATION

1. To understand the gross features of nuclear systems from a QCD perspective.

2. To develop a new approach for the construction of effective interactions suitable for NCSM calculations, which avoids uncontrolled approximations and the use of internucleon potentials.

No Core Shell Model

"*Ab Initio*" approach to microscopic nuclear structure calculations, in which all A nucleons are treated as being active.

Want to solve the A-body Schrödinger equation

$$H_A \Psi^A = E_A \Psi^A$$

R P. Navrátil, J.P. Vary, B.R.B., PRC 62, 054311 (2000)

P. Navratil, et al., J. Phys. G: Nucl. Part. Phys. 36, 083101 (2009)

From few-body to many-body

Ab initio
No Core Shell Model

Realistic NN & NNN forces

↓

Effective interactions in cluster approximation

↓

Diagonalization of many- body Hamiltonian

↓

Many-body experimental data

GOAL

The formulation of an Effective Field Theory (EFT) with only nucleon fields directly in the NCSM model space.

OUTLINE

I. Brief Overview of Effective Field Theory (EFT)

II. Formulation of the NCSM in an EFT Framework

III. Summary and Outlook

I. Brief Overview of Effective Field Theory (EFT)

Effective Field Theory (1/3)

i) Separation of scale :

M_{QCD} ~ 1 GeV (mass of nucleon)

M_{nucl} ~ 100 MeV (typical momentum in a nucleus)

M_{struct} ~ 10 MeV (binding energy of a nucleon in a nucleus)

-> details of physics at short distance (high energy) are irrelevant for low energy physics.

-> in EFT low energy degrees of freedom are explicitly included (high momenta are integrated out).

ii) The Lagrangian / potential consistent with symmetries is expanded as a Taylor Series:

$$V(\vec{p}\,', \vec{p}) = \sum_{i,j} C_{i,j} (\vec{p})^i (\vec{p}\,')^j$$

Effective Field Theory (2/3)

iii) Regularization and renormalization :

-> cut-off Λ (separation between low and high energy physics)

$$V(\vec{p}\,',\vec{p}) => \sum_{i,j} C_{i,j}(\Lambda)(\vec{p})^i(\vec{p}\,')^j$$

-> no dependence on cut-off for observables (for a high enough cut-off), dependence absorbed by coupling constants (fitted with observables).

Effective Field Theory (3/3)

iv) Find the power counting (“truncation of the Taylor series”):

-> hierarchy between the different contributions

-> results improvable order by order (Leading Order, Next-to-Leading-Order, Next-to-Next-to-Leading-Order......)

II. The Formulation of the NCSM in an EFT Framework

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ScienceDirect
PHYSICS LETTERS B
Physics Letters B 653 (2007) 358–362
www.elsevier.com/locate/physletb

No-core shell model in an effective-field-theory framework

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Abstract

We present a new approach to the construction of effective interactions suitable for many-body calculations by means of the no-core shell model (NCSM). We consider an effective field theory (EFT) with only nucleon fields directly in the NCSM model spaces. In leading order, we obtain the strengths of the three contact interactions from the condition that in each model space the experimental ground-state energies of ^{2}H, ^{3}H and ^{4}He be exactly reproduced. The first $(0^{+};0)$ excited state of ^{4}He and the ground state of ^{6}Li are then obtained by means of NCSM calculations in several spaces and frequencies. After we remove the harmonic-oscillator frequency dependence, we predict for ^{4}He an energy level for the first $(0^{+};0)$ excited state in remarkable agreement with the experimental value. The corresponding ^{6}Li binding energy is about 70% of the experimental value, consistent with the expansion parameter of the EFT.

PACS: 21.30.-x; 21.60.Cs; 24.10.Cn; 45.50.Jf

J. Phys. G: Nucl. Part. Phys. **37** (2010) 064033 (11pp) doi:10.1088/0954-3899/37/6/064033

Effective interactions for light nuclei: an effective (field theory) approach

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Abstract

One of the central open problems in nuclear physics is the construction of effective interactions suitable for many-body calculations. We discuss a recently developed approach to this problem, where one starts with an effective field theory containing only fermion fields and formulated directly in a no-core shell-model space. We present applications to light nuclei and to systems of a few atoms in a harmonic-oscillator trap. Future applications and extensions, as well as challenges, are also considered.

PHYSICAL REVIEW C **85**, 034003 (2012)

Two and three nucleons in a trap, and the continuum limit

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We describe systems of two and three nucleons trapped in a harmonic-oscillator potential with interactions from the pionless effective field theory up to next-to-leading order (NLO). We construct the two-nucleon interaction using two-nucleon scattering information. We calculate the trapped levels in the three-nucleon system with isospin $T = 1/2$ and determine the three-nucleon force needed for stability of the triton. We extract neutron-deuteron phase shifts, and show that the quartet scattering length is in good agreement with experimental data.

DOI: 10.1103/PhysRevC.85.034003 PACS number(s): 21.30.Fe, 21.45.−v

Why EFT + NCSM?

EFT:

1. Captures the relevant degrees of freedom/symmetries
2. Builds in the correct long-range behavior
3. Has a systematic way for including the short-range behavior/order by order
4. Many-body and two-body interactions treated in the same framework
5. Explains naturally the hierarchy of the (many-body) forces

NCSM:

1. Flexible many-body method/easy to implement
2. Equivalent SD and Jacobi formulations
3. Can handle both NN and NNN interactions
4. In principle applies to any nucleus/extensions to heavier nuclei

Pionless EFT for nuclei within the NCSM:

Without pions--> Breakdown momentum roughly 100 MeV/c

The Hamiltonian at Leading Order has 2 NN contact interactions in the triplet S_1 and singlet S_0 channels and a 3-body contact interaction in the 3-nucleon S_1/2 channel.

The Schroedinger equation is solved for this Hamiltonian in the NCSM basis space of size N_max and the coupling constants are fitted to the binding energies of the deuteron, triton and the alpha particle.

$$H = \frac{1}{2m_N A} \sum_{[i<j]} (\vec{p}_i - \vec{p}_j)^2 + C_0^1 \sum_{[i<j]^1} \delta(\vec{r}_i - \vec{r}_j) + C_0^0 \sum_{[i<j]^0} \delta(\vec{r}_i - \vec{r}_j) + D_0 \sum_{[i<j<k]} \delta(\vec{r}_i - \vec{r}_j)\delta(\vec{r}_j - \vec{r}_k),$$

Stetcu et. al., 2007

PLB 653, pp. 358-362

CUTOFFS

1. Ultraviolet Cutoff: Want convergence as Lamda increases.

$$\Lambda = \sqrt{m_N (N_{\max} + 3/2)\hbar\omega}$$

2. Infrared Cutoff: Want convergence as omega decreases.

$$\lambda = \sqrt{m_N \hbar\omega}$$

Results for the first excited state of the alpha particle: I. Stetcu, et al., Phys. Lett. B 653, 358 (2007)

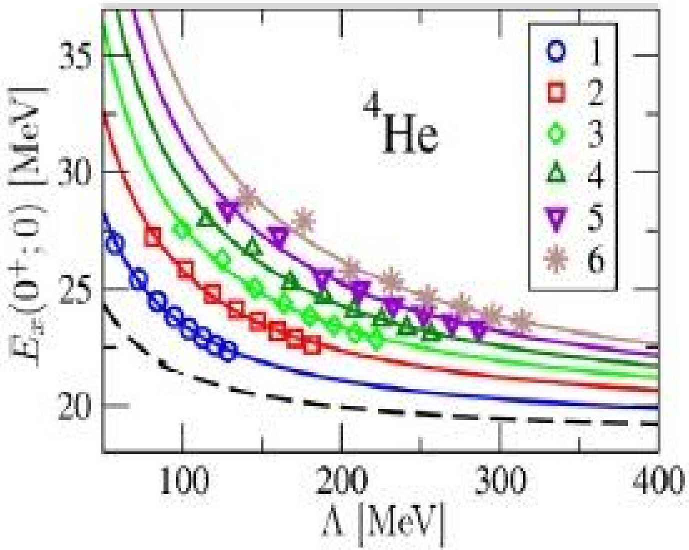


Fig. 2. Dependence of the first $(0^+;0)$ excitation energy in ^{4}He on the ultraviolet momentum cutoff Λ. For each frequency $\hbar\omega$, given in the legend in MeV, we interpolate the direct results (discrete symbols) with a $1/\Lambda$ dependence (continuous curves). The dashed curve marks the limit $\hbar\omega \to 0$.

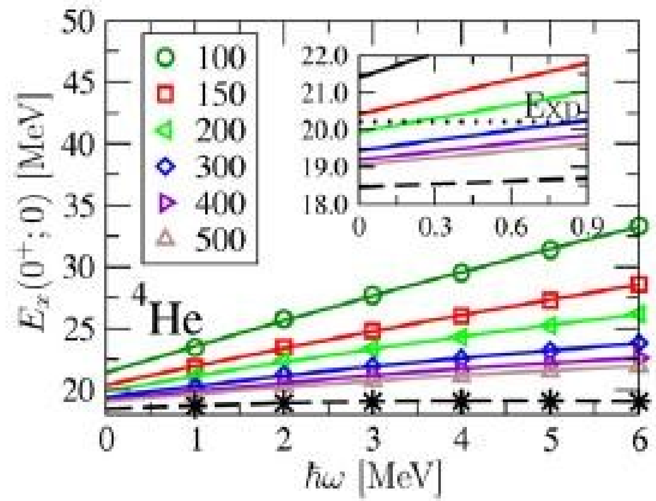


Fig. 3. Dependence of the first $(0^+;0)$ excitation energy in ^{4}He on the infrared energy cutoff $\hbar\omega$. For each ultraviolet cutoff Λ, given in the legend in MeV, we interpolate as described in the text. The results marked with star symbols are obtained in the limit $\Lambda \to \infty$. In the insert we show the variation around the origin, compared to the experimental value.

E(theory)= 18.5 Mev; E(experiment) = 20.21 MeV: Agree within 10% in LO.

Running of the Coupling Constants with Lambda and omega

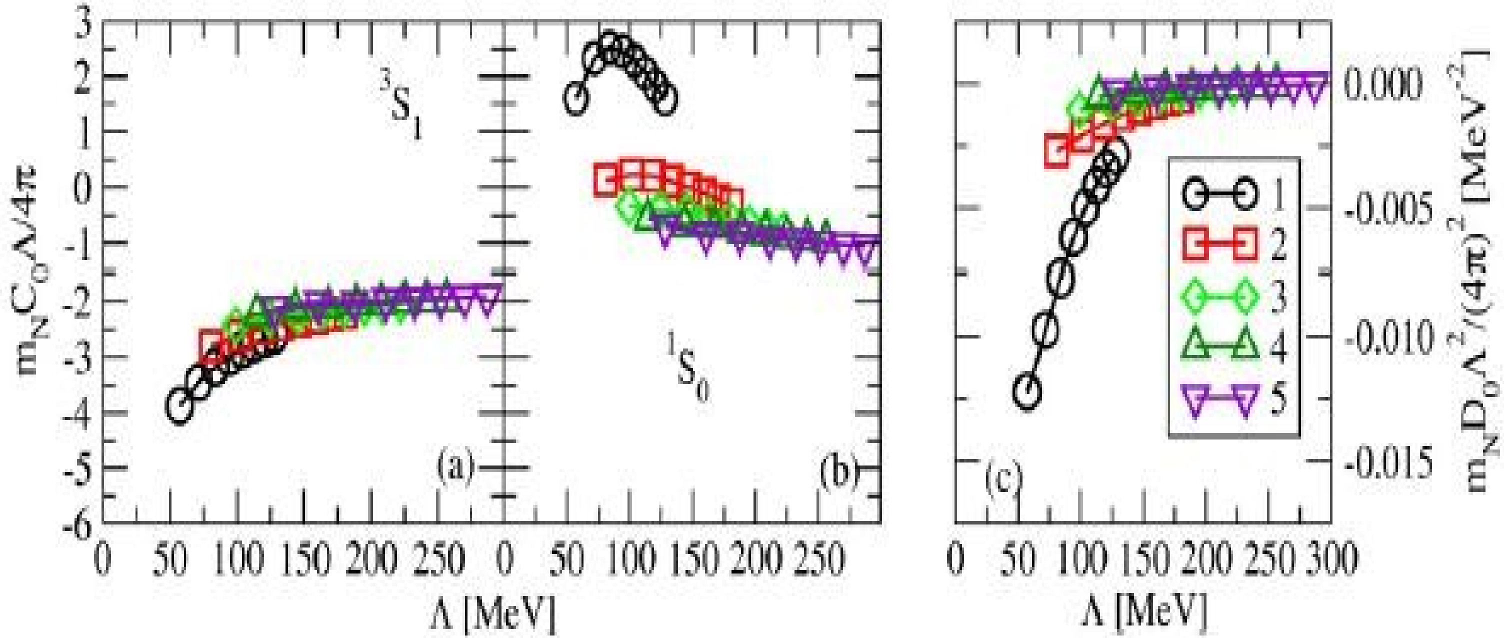


Fig. 1. Running of the scaled NN and NNN coupling constants with the ultraviolet cutoff Λ at various frequencies $\hbar\omega$ given in the legend in MeV: (a) $m_N \Lambda C_0^1/4\pi$; (b) $m_N \Lambda C_0^0/4\pi$; and (c) $m_N \Lambda^2 D_0/(4\pi)^2$.

Difficulties:

fixing the couplings to few-body states is cumbersome

HO: bound states only

no immediate connection to the scattering observables

Question : How to construct an EFT within a bound many-body model space beyond **Leading-Order** ?

Answer : by trapping nuclei in a harmonic potential

T. Busch, et al., Found. Phys. 28, 549 (1998)

$$\boxed{\frac{\Gamma\left(\frac{3}{4}-\frac{E}{2\hbar\omega}\right)}{\Gamma\left(\frac{1}{4}-\frac{E}{2\hbar\omega}\right)} = -\frac{bk}{2}\cot\delta}$$

energy in the trap (bound state physics)

phase shift (scattering physics)

$$k\cot\delta = -\frac{1}{a_2} + \frac{1}{2}r_2k^2 + \ldots,$$

Effective Range Expansion

THE TRAP AS GENUINE INFRARED CUTOFF

$$H_{int} = \frac{1}{A}\sum_{i>j=1}^{A}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m} + \sum_{i>j=1}^{A} V_{ij} + \sum_{i>j>k=1}^{A} V_{ijk} + \ldots$$

$$H_{int} + H_A^{trap} = \frac{1}{A}\sum_{i>j=1}^{A}\left(\frac{(\vec{p}_i-\vec{p}_j)^2}{2m} + \frac{1}{2}m\omega^2(\vec{r}_i-\vec{r}_j)^2\right) + \sum_{i>j=1}^{A} V_{ij} + \ldots$$

- trap: renormalization of the interaction
- solve the many-body problem in the trap
- take the limit $\omega \to 0$

EFT for Two Particles in a Trap

At the heart of an effective theory: a truncation of the Hilbert space / all interactions allowed by symmetries are generated / power counting

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)}=\frac{b}{2a_2}$$

$$\frac{\Gamma(3/4-\varepsilon/2)}{\Gamma(1/4-\varepsilon/2)}=-\frac{b}{2}\left(-\frac{1}{a_2}+\frac{r_2}{b^2}\varepsilon+\ldots\right)$$

In finite model spaces:

$$V_{LO}(\vec{p},\vec{p}')=C_0$$

$$V_{NLO}(\vec{p},\vec{p}')=C_2(p^2+p'^2)$$

$$V_{N^2LO}(\vec{p},\vec{p}')=C_4(p^2+p'^2)^2$$

$C_0, C_2, C_4, \ldots$

Constants to be determined in each model space so that select observables are preserved

LO Renormalization:

$$\Psi(\vec{r}) = \sum_{n=0}^{N_{\max}/2} A_n \varphi_n(\vec{r})$$

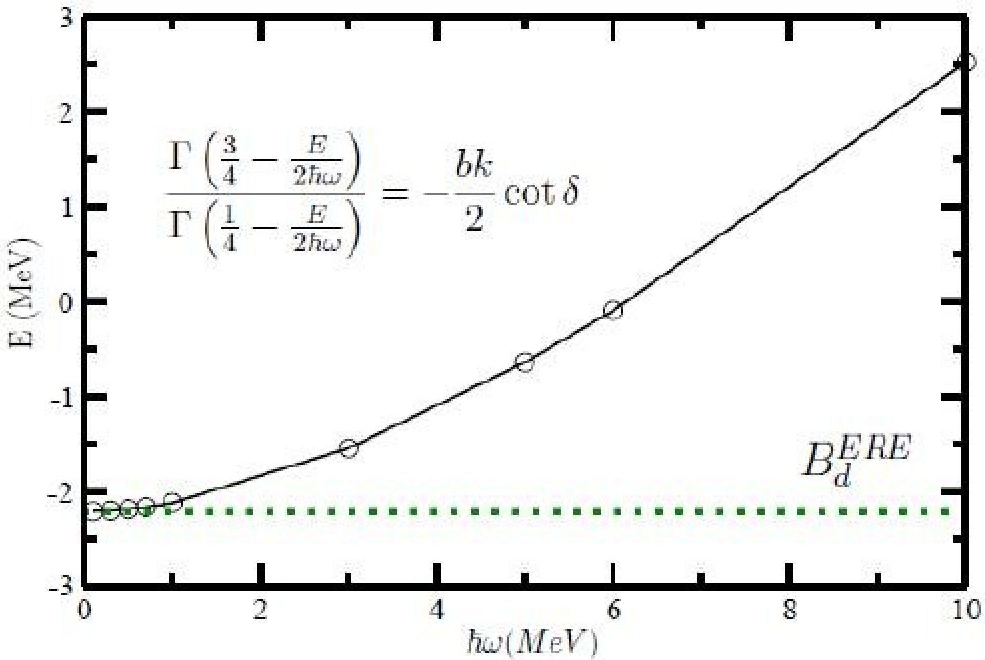
$$\left[b^2 p^2 + \frac{r^2}{b^2} + 2\mu C_0(N_{\max}) b^2 \delta^{(3)}(\vec{r}) \right] \Psi(\vec{r}) = 2\frac{E}{\omega} \Psi(\vec{r})$$

$$\frac{1}{C_0(N_{\max})} = -\sum_{n=0}^{N_{\max}/2} \frac{|\varphi_n(0)|^2}{2n + 3/2 - \varepsilon}$$

Fix from Busch's formula

How far can we go in trapping the system to describe intrinsically untrapped physics *i.e.* free nuclei ?

Energy of a "trapped deuteron"



Binding energy of a free deuteron

$$k\ \cot\delta = -\frac{1}{a_2} + \frac{1}{2}r_2k^2$$

$$ik + k\cot\delta = 0$$

$$a_t = 5,425fm \quad r_t = 1.75fm$$

B_d^{ERE} ~-2,221 MeV

=> ω should be as small as possible

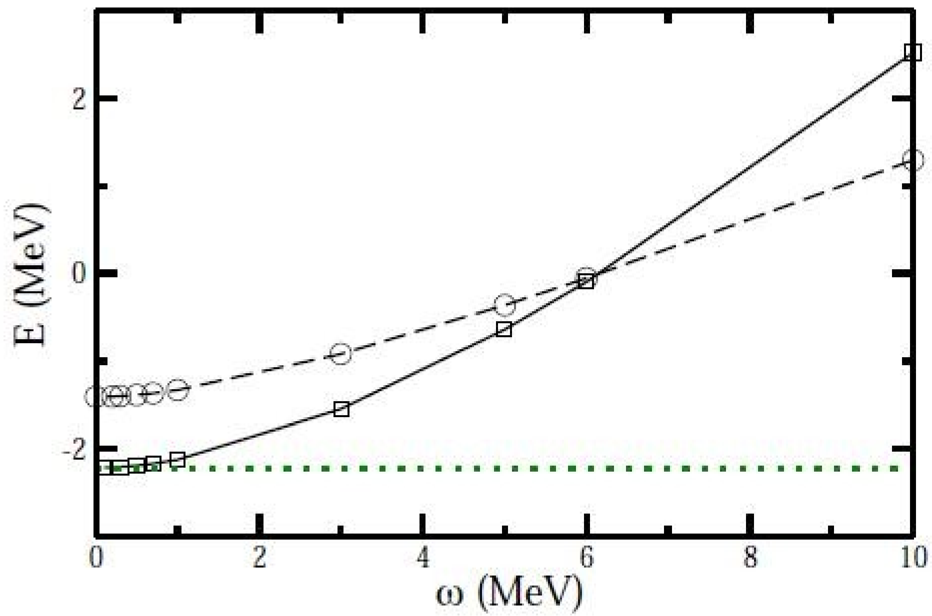


FIG. 1: Ground-state energy of the trapped two-nucleon system in the $^3S_1$ channel (deuteron in the trap) as a function of the frequency ω. The energy at LO (NLO) is given by the dashed (solid) line. For small values of ω, the energy converges to the value in free space, which is, at NLO, indicated by the dotted line.

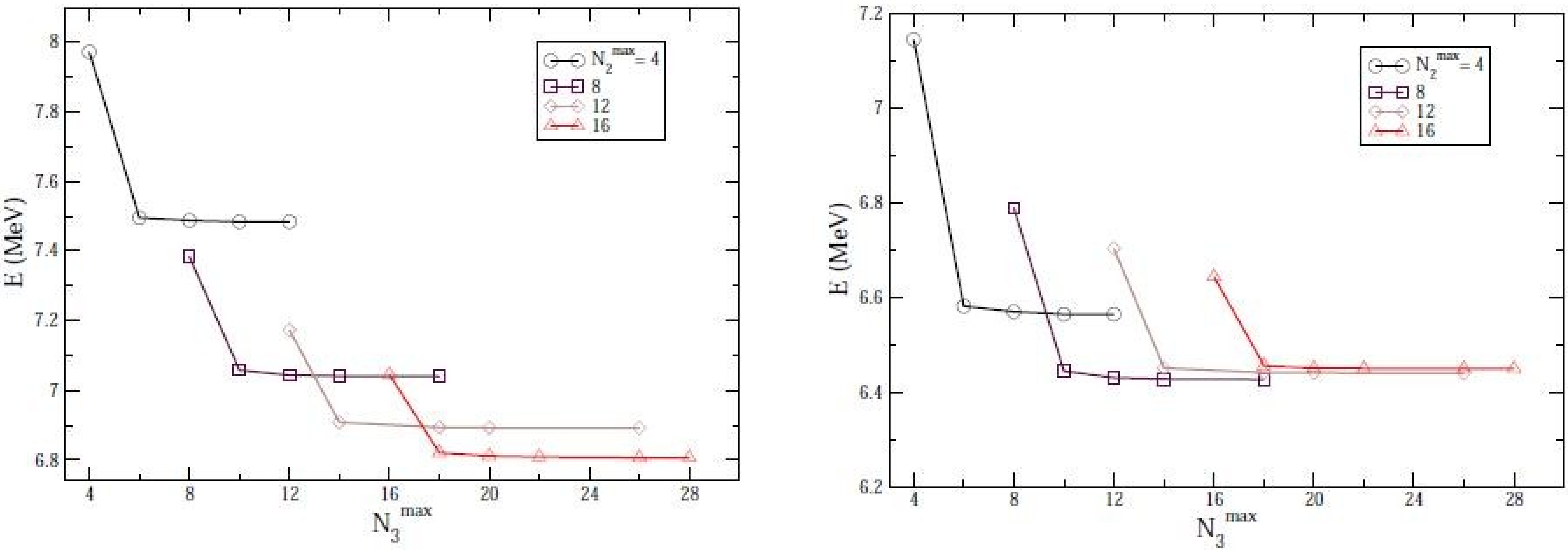
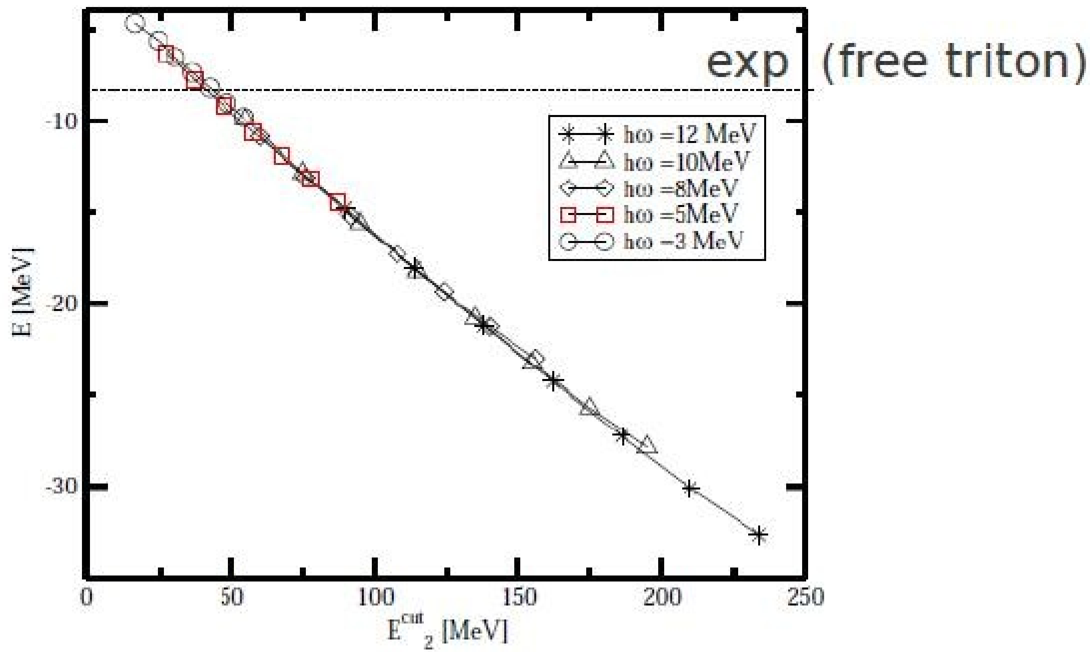


FIG. 6: Ground-state energy of the trapped three-nucleon system coupled to $T = 1/2$, $J^\pi = 3/2^+$ as function of the three-body model-space size N_3^{max}, for $\omega = 3$ MeV: LO (left panel) and NLO (right panel). Results are shown for different values of the two-body model-space size N_2^{max}.

Binding energy of a trapped triton at Leading Order

$$T = 1/2 \text{ and } J^{\pi} = 1/2^{+}$$



-> the 3-nucleon system collapses as the two-(three)body cutoff is increased (Thomas effect)

-> need for a three-body force at LO (as in the continuum)

3. SUMMARY AND OUTLOOK

SUMMARY

1. Formulation of EFT principles directly into the NCSM model space.

2. Renormalization of the interaction is intimately related with the model space used to solve the many-body problem.

3. Applications: Results for 2- and 3-body nuclei in good agreement with experiment. Reasonable agreement for 4- and 6-body nuclei.

OUTLOOK

1. Extension to pionful theory, loosely bound nuclei, scattering observables, heavier nuclei, etc.

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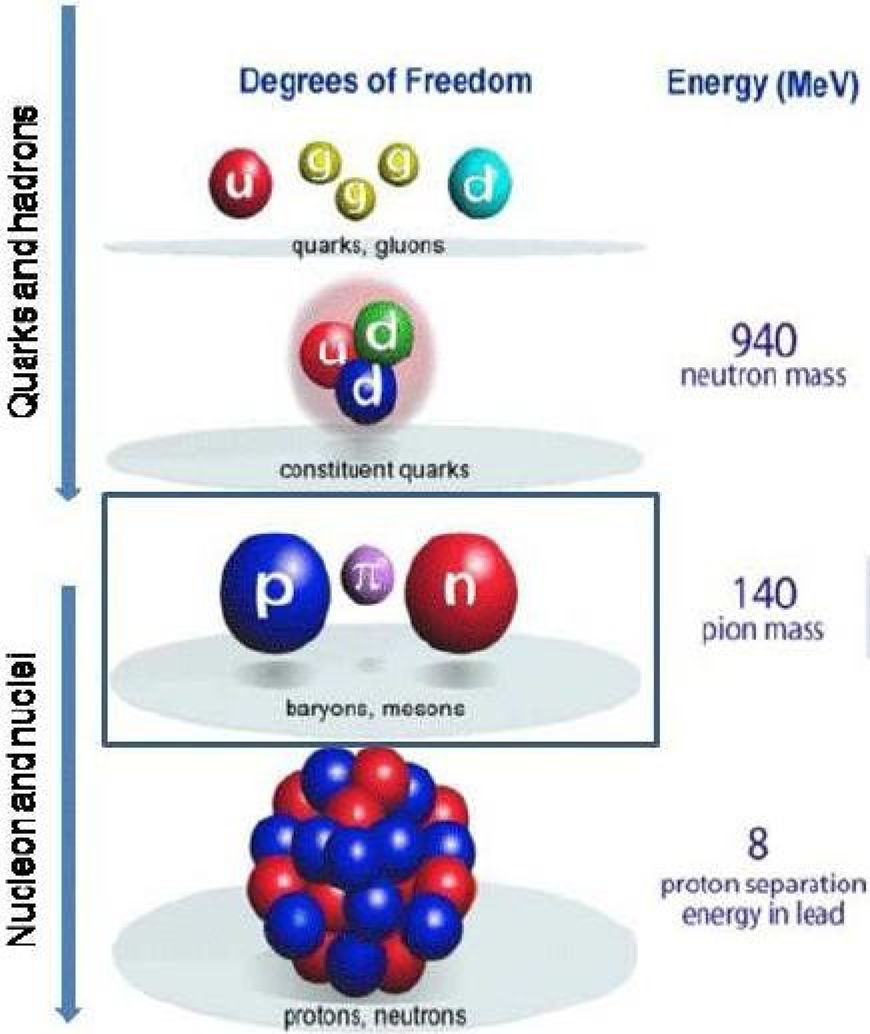
Construction of an Effective Field Theory



i) Identify the relevant degrees of freedom :

-> details of physics at short distance are irrelevant for low energy physics, high-energy degrees of freedom are integrated out.

ii) Construct the most general potential/Lagrangian consistent with the symmetries of the system

iii) Design an organizational principle (power counting) that can distinguish between more or less important contributions.

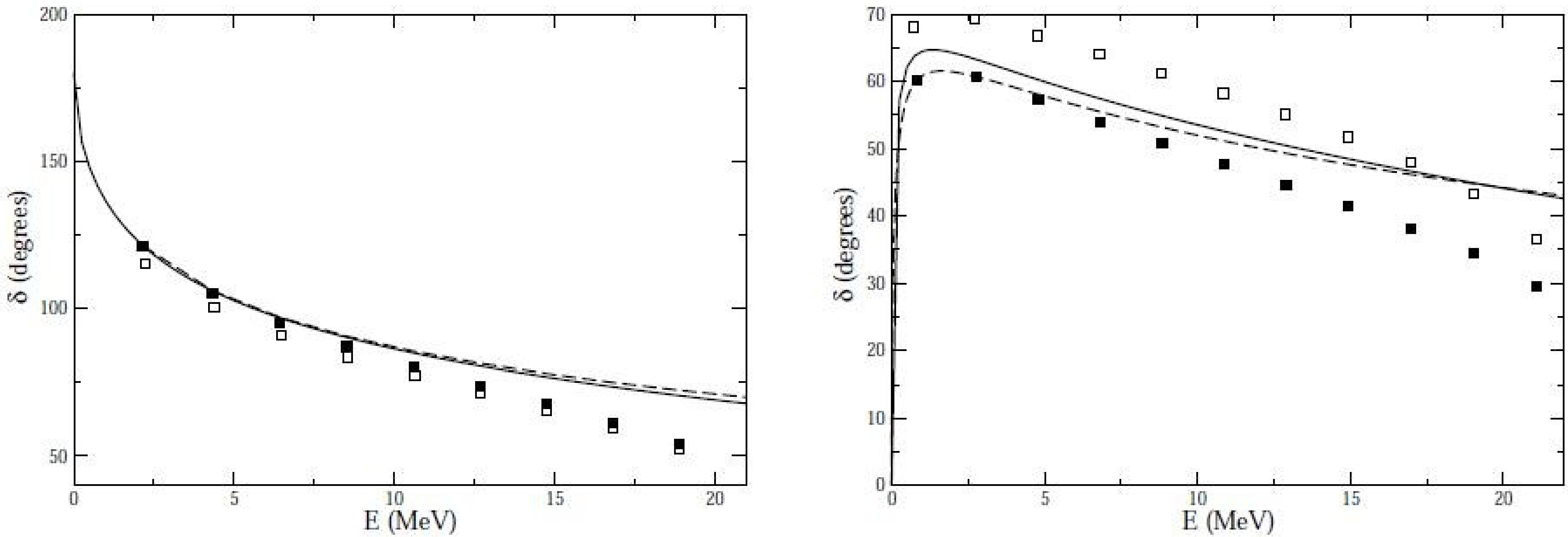


FIG. 5: Phase shifts for the two-nucleon system at $\omega = 1$ MeV and $N_{max} = 20$ as a function of the relative energy: $^3S_1$ (left panel) and $^1S_0$ (right panel). EFT results at LO (NLO) are marked by empty (filled) squares; the ERE up to the effective range is indicated by a dashed line; and the Nijmegen np PSA [20] by a solid line.

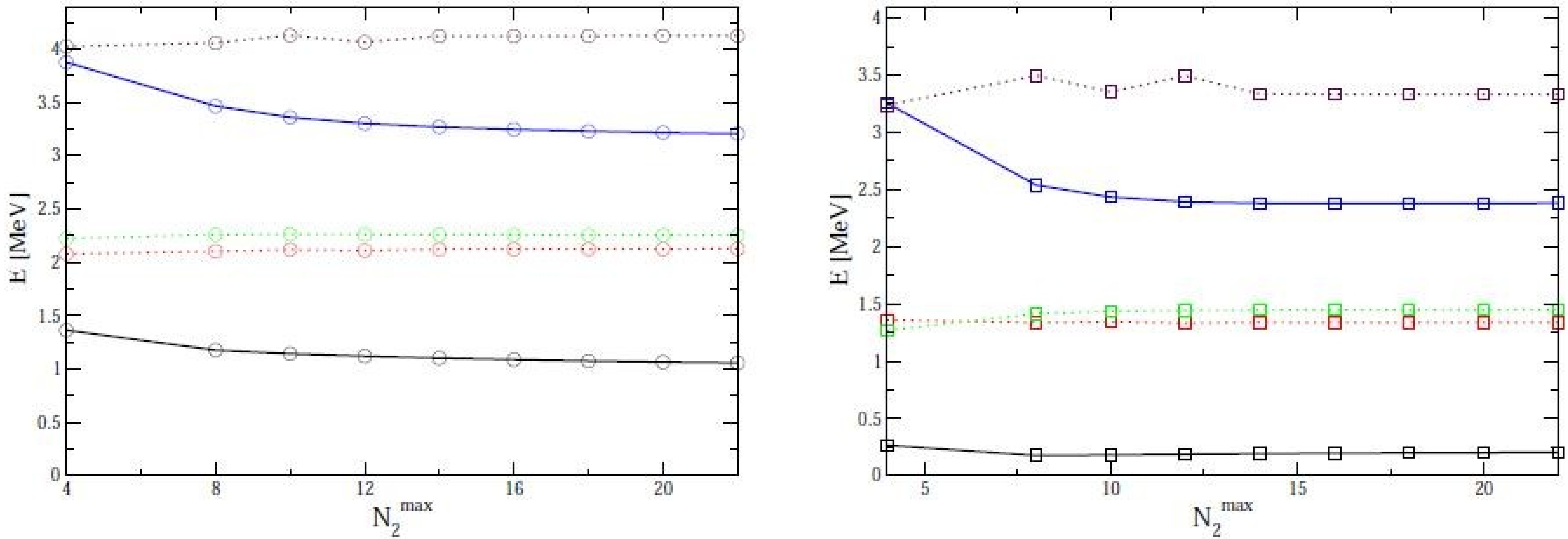


FIG. 8: Lowest energies of the trapped three-nucleon system with $T = 1/2$, $J^\pi = 3/2^+$ as a function of N_2^{max}, for $\omega = 1$ MeV: LO (left panel) and NLO (right panel). The ground state and the third excited state (full lines) correspond to neutron-deuteron scattering within the trap in the $L = 0$, $S = 3/2$ channel, whereas the other states shown correspond to different L, S configurations.

$$H\Psi_\alpha = E_\alpha \Psi_\alpha \quad \text{where} \quad H = \sum_{i=1}^{A} t_i + \sum_{i\leq j}^{A} v_{ij}.$$

$$\mathcal{H}\Phi_\beta = E_\beta \Phi_\beta$$

$$\Phi_\beta = P\Psi_\beta$$

P is a projection operator from S into $\mathcal{S}$

$$< \tilde{\Phi}_\gamma | \Phi_\beta >= \delta_{\gamma\beta}$$

$$\mathcal{H} = \sum_{\beta \epsilon \mathcal{S}} |\Phi_\beta > E_\beta < \tilde{\Phi}_\beta|$$

Effective Hamiltonian for NCSM

Solving $\boxed{H^{\Omega}_{A,\,a=2}\Psi_{a=2} = E^{\Omega}_{A,\,a=2}\Psi_{a=2}}$ in "infinite space" 2n+l = 450 relative coordinates

P + Q = 1; P – model space; Q – excluded space;

$$\boxed{E^{\Omega}_{A,2} = U_2 H^{\Omega}_{A,2} U_2^{\dagger}} \quad U_2 = \begin{pmatrix} \boxed{U_{2,P}} & U_{2,PQ} \\ U_{2,QP} & U_{2,Q} \end{pmatrix} \quad E^{\Omega}_{A,2} = \begin{pmatrix} \boxed{E^{\Omega}_{A,2,P}} & 0 \\ 0 & E^{\Omega}_{A,2,Q} \end{pmatrix}$$

$$\boxed{H^{N_{\max},\Omega,\mathrm{eff}}_{A,2} = \frac{U^{\dagger}_{2,P}}{\sqrt{U^{\dagger}_{2,P}U_{2,P}}} E^{\Omega}_{A,2,P} \frac{U_{2,P}}{\sqrt{U^{\dagger}_{2,P}U_{2,P}}}}$$

Two ways of convergence:

1) For P $\longrightarrow$ 1 and fixed a: $H^{eff}_{A,a=2} \longrightarrow H_A$

2) For a $\longrightarrow$ A and fixed P: $H^{eff}_{A,a} \longrightarrow H_A$