

# *Ab Initio* Widths and Asymptotic Normalizations

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Nollett & Wiringa, PRC 83, 041001(R) (2011)  
and Nollett, PRC in press, arXiv:1206.0046

## General orientation

*Ab initio* nuclear calculations have had great success over the past 15 years

Nuclear structure does indeed have quantitative roots in the vacuum NN interaction

Several features of light-nucleus energy spectra are reproduced:

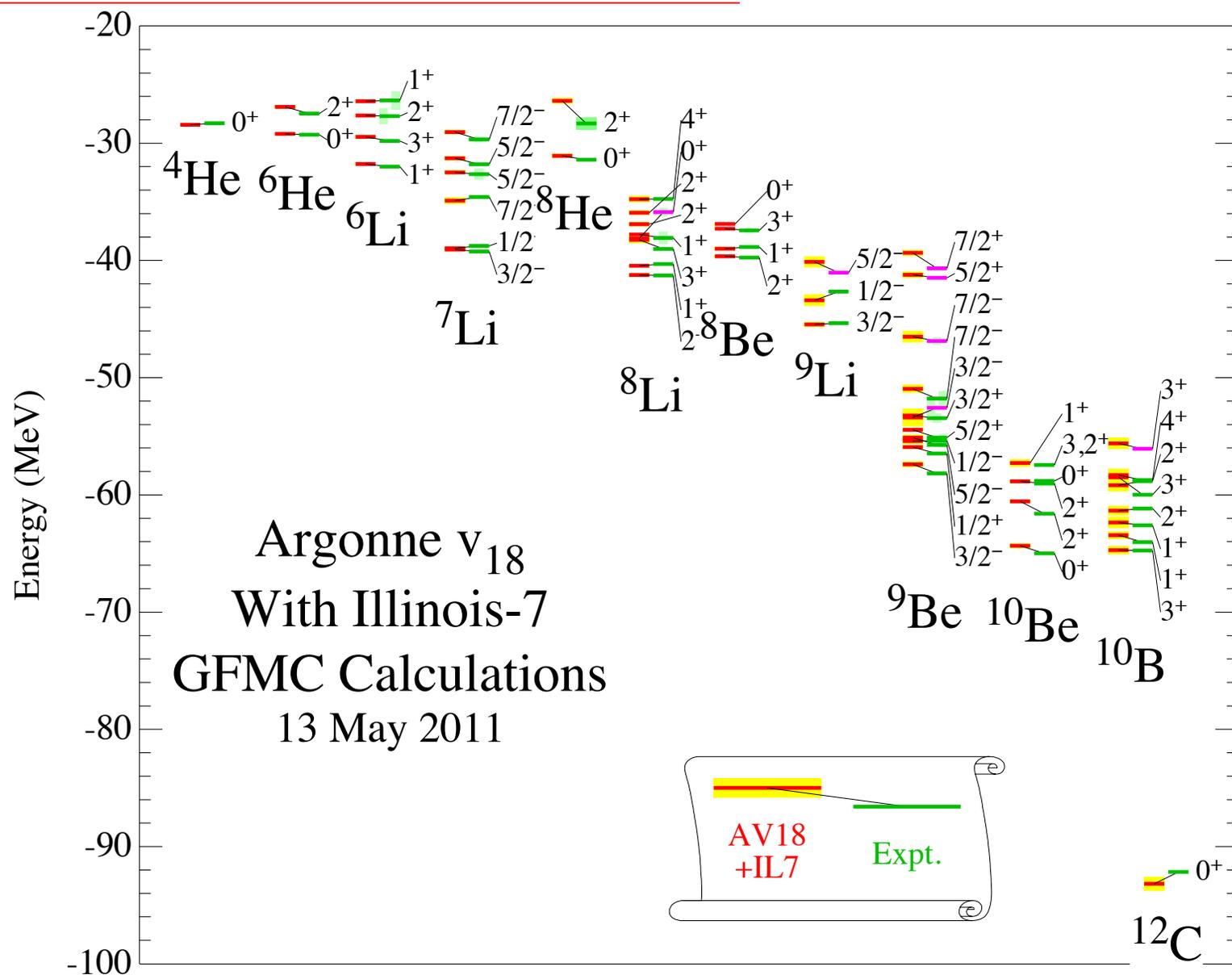
- Overall scale of binding energy
- Orderings of  $J^\pi$  states (including  $^{10}\text{B}$  ground state, sensitive to NNN force)
- Spin-orbit splittings (also sensitive to NNN)

Quantum Monte Carlo achieved some of these things first

QMC dependence on  $A$  is steeper than Moore's Law, so it may not go beyond  $A = 12$  without some big change to algorithms

Many things remain to be done at  $A \leq 12$ : reactions, transitions, overlaps, etc.

## A sampling of the quantum Monte Carlo results



## Overlaps, tails, widths, and phase shifts

Several goals for the near future are served by a single tool: integral relations between the short-range wave function and long-range properties

Many narrow unbound states on the previous page were computed as if bound  
– how to get widths from those?

The wave functions converge rapidly at short range but poorly at long range  
– how to remedy that?

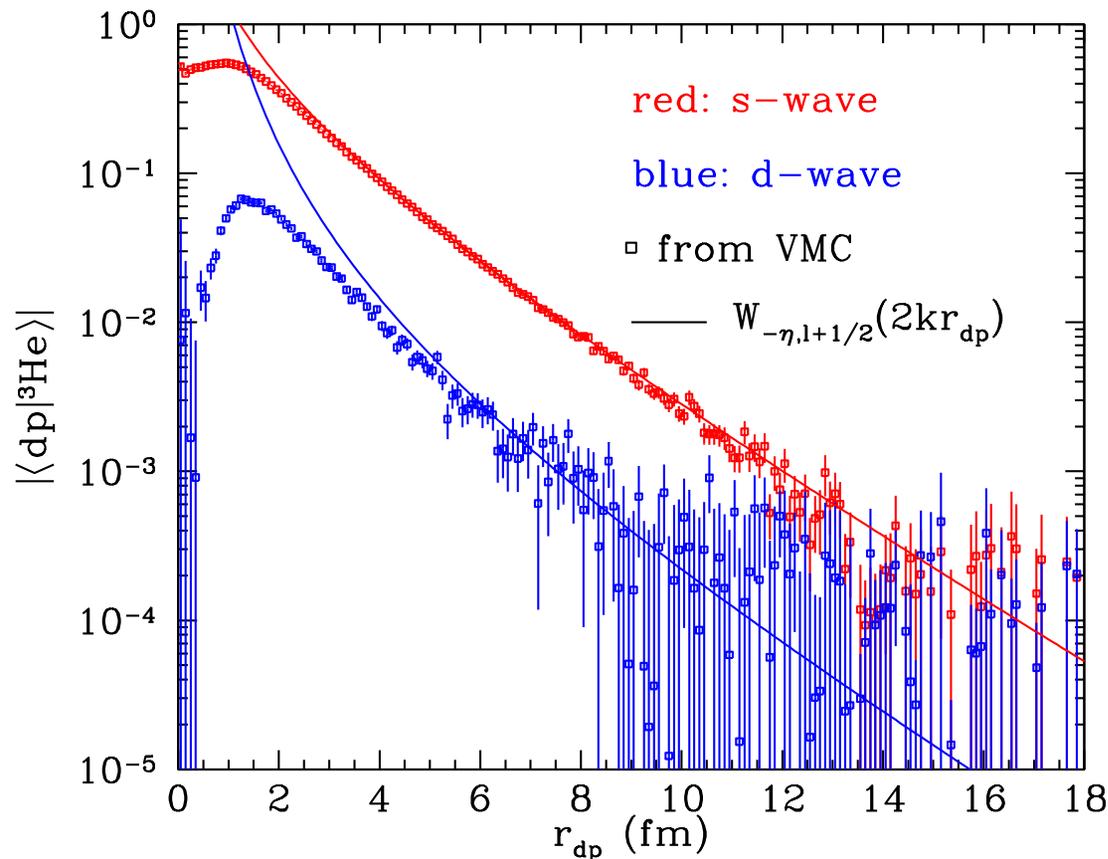
Overlaps are needed for transfer/knockout/breakup reactions  
– how to compute them accurately?

Cross sections naïvely require laborious computation of solutions at many boundary conditions  
– is there a less labor-intensive approximation?

## Asymptotic normalization coefficient (ANC): definition

Many-body wave functions at large cluster separations factorize into clusters times a known shape:

$$\Phi_{3\text{He}}(r_{pd} \rightarrow \infty) = \sum_{l=0,2} C_{lj} \phi_d \phi_p Y_{lm}(\hat{\mathbf{r}}_{pd}) W_{-\eta, l+1/2}(2kr_{pd})/r_{pd}$$



At long range, nuclear dynamics just set  $E$  ( $\rightarrow \eta, k$ ) and  $C_{lj}$

## Why compute and measure ANCs?

ANCs are useful for estimating low-energy direct-capture cross sections like  ${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma$  that are hard to measure directly

ANCs are observables in the quantum-mechanical sense (they're residues of  $S$ -matrix poles), unlike spectroscopic factors

ANCs are computable and must agree between phase-equivalent potentials (unlike spectroscopic factors)

ANCs are as real as charge or matter radii and are equally valid tests of theory

Relatively few ANCs have been measured  $\longrightarrow$  an opportunity for pre- rather than post-diction

ANCs provide a learning problem for integral relations

## Integral relation for the ANC

Wave function tails are hard to compute accurately, but we can learn the ANC without computing the tail (goes back to 1970s)

The Schrödinger equation

$$(H - E) \Psi_A = 0$$

may be separated into parts internal to  $\Psi_{A-1}$  and parts involving the last particle (distance  $r_{cc}$  away) to yield

$$\Psi_A = - [T_{\text{rel}} + V_C + B]^{-1} (U_{\text{rel}} - V_C) \Psi_A$$

which implies

$$C_{lj} = \frac{2\mu}{k\hbar^2 w} A \int \frac{M_{-\eta, l+\frac{1}{2}}(2kr_{cc})}{r_{cc}} \Psi_{A-1}^\dagger \chi^\dagger Y_{lm}^\dagger(\hat{\mathbf{r}}_{cc}) (U_{\text{rel}} - V_C) \Psi_A d\mathbf{R}$$

$M_{-\eta, l+\frac{1}{2}}(2kr)$  is the “other” Whittaker function, irregular at  $r \rightarrow \infty$ ,

and  $\mathbf{R} = (\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$ , with  $r_{cc} = r_A - \frac{1}{A-1} \sum_{i=1}^{A-1} \mathbf{r}_i$

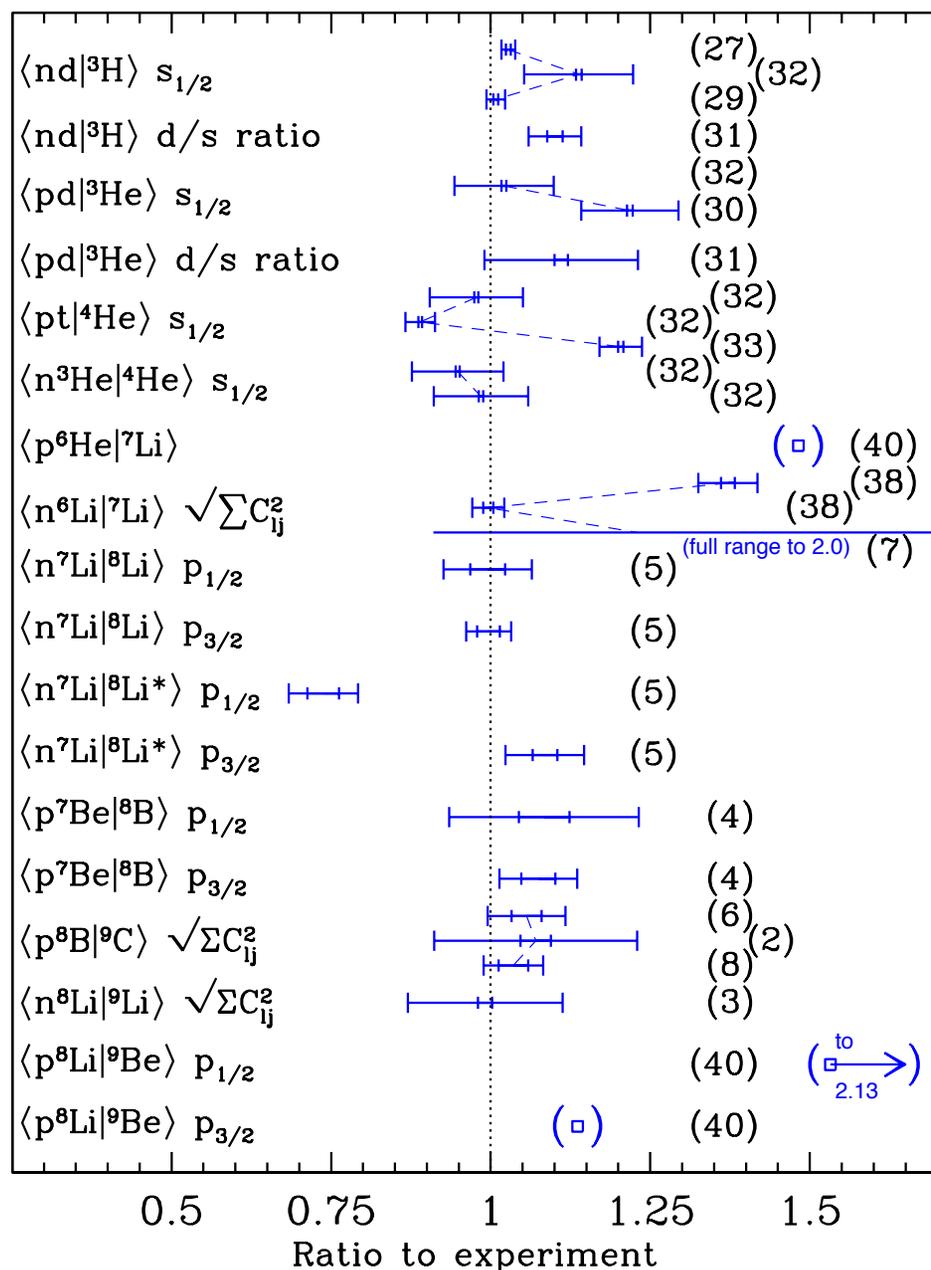
## The results, $3 \leq A \leq 9$ one-nucleon removal, just VMC (not GFMC)

$A$	$A - 1$	$s_{1/2}$	$d_{3/2}$	$C_{d3/2}/C_{s1/2}$	
${}^3\text{H}$	${}^2\text{H}$	2.127(8)	-0.0979(9)	-0.0460(5)	
${}^3\text{He}$	${}^2\text{H}$	2.144(8)	-0.0927(10)	-0.0432(5)	
${}^4\text{He}$	${}^3\text{H}$	-6.55(2)			
${}^4\text{He}$	${}^3\text{He}$	6.42(2)			
$A$	$A - 1$	$p_{1/2}$	$p_{3/2}$	$f_{5/2} \times 10^3$	$f_{7/2} \times 10^3$
${}^7\text{Li}$	${}^6\text{He}$		3.68(5)		
${}^7\text{Li}^*$	${}^6\text{He}$	3.49(5)			
${}^7\text{Li}$	${}^6\text{Li}$	1.652(12)	1.890(13)	-78(20)	
${}^7\text{Li}^*$	${}^6\text{Li}$	-0.543(16)	-2.54(4)		
${}^7\text{Be}$	${}^6\text{Li}$	-1.87(3)	-2.15(3)	63(9)	
${}^7\text{Be}^*$	${}^6\text{Li}$	0.559(16)	2.59(5)		
${}^8\text{Li}$	${}^7\text{Li}$	0.218(6)	-0.618(11)	5.2(5)	2.5(15)
${}^8\text{Li}^*$	${}^7\text{Li}$	-0.090(3)	0.281(5)	-0.6(2)	
${}^8\text{B}$	${}^7\text{Be}$	0.246(9)	-0.691(17)	1.1(2)	-1.1(5)
${}^9\text{C}$	${}^8\text{B}$	-0.309(7)	1.125(12)	1.9(5)	-0.5(18)
${}^9\text{Li}$	${}^8\text{Li}$	0.308(7)	-1.140(13)	-4.1(10)	5(3)
${}^9\text{Li}$	${}^8\text{Li}^*$	-0.122(3)	0.695(7)	-1.1(6)	
${}^9\text{Li}$	${}^8\text{He}$		-5.99(8)		
${}^9\text{Be}$	${}^8\text{Li}$	5.03(6)	9.50(11)	35(34)	257(112)
${}^9\text{Be}$	${}^8\text{Li}^*$	6.56(5)	-6.21(7)	364(40)	

Nollett & Wiringa, PRC 83, 041001(R) (2011)

The small  $f$ -wave amplitudes are accessible with this method – unknown how reliable (or measurable), but something new

## Readable results, where there are “experimental” data



Small error bars are VMC statistics

Large ones are “experimental”

Sensitivity to wave function construction seems weak but hard to quantify

$A \leq 4$  clearly dominated by systematics, also old

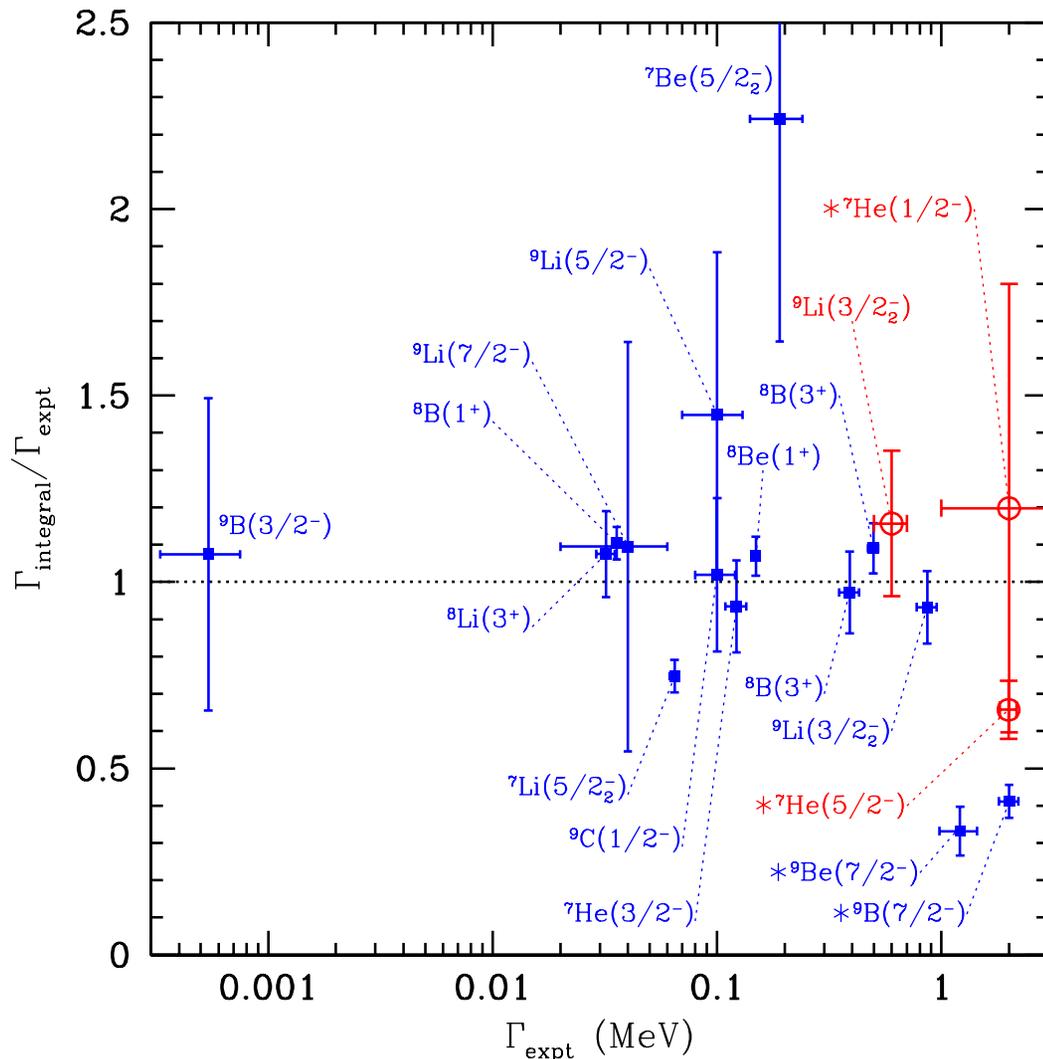
With a couple of exceptions, these are the first *ab initio* ANCs in  $A > 4$

In a capture model, our  $^8B$  ANCs give  $S_{17}(0) = 20.8 \text{ eV} \cdot \text{b}$ , same as recommended value

## Widths as squared ANCs of resonant states

Widths are proportional to ANCs of resonant states,  $\Gamma \simeq \frac{\hbar^2 k}{\mu} |C_{lj}|^2$

I've chosen low-lying states in  $A \leq 9$  with width mainly/all in nucleon emission



Red: overlaps inconsistent with resonance

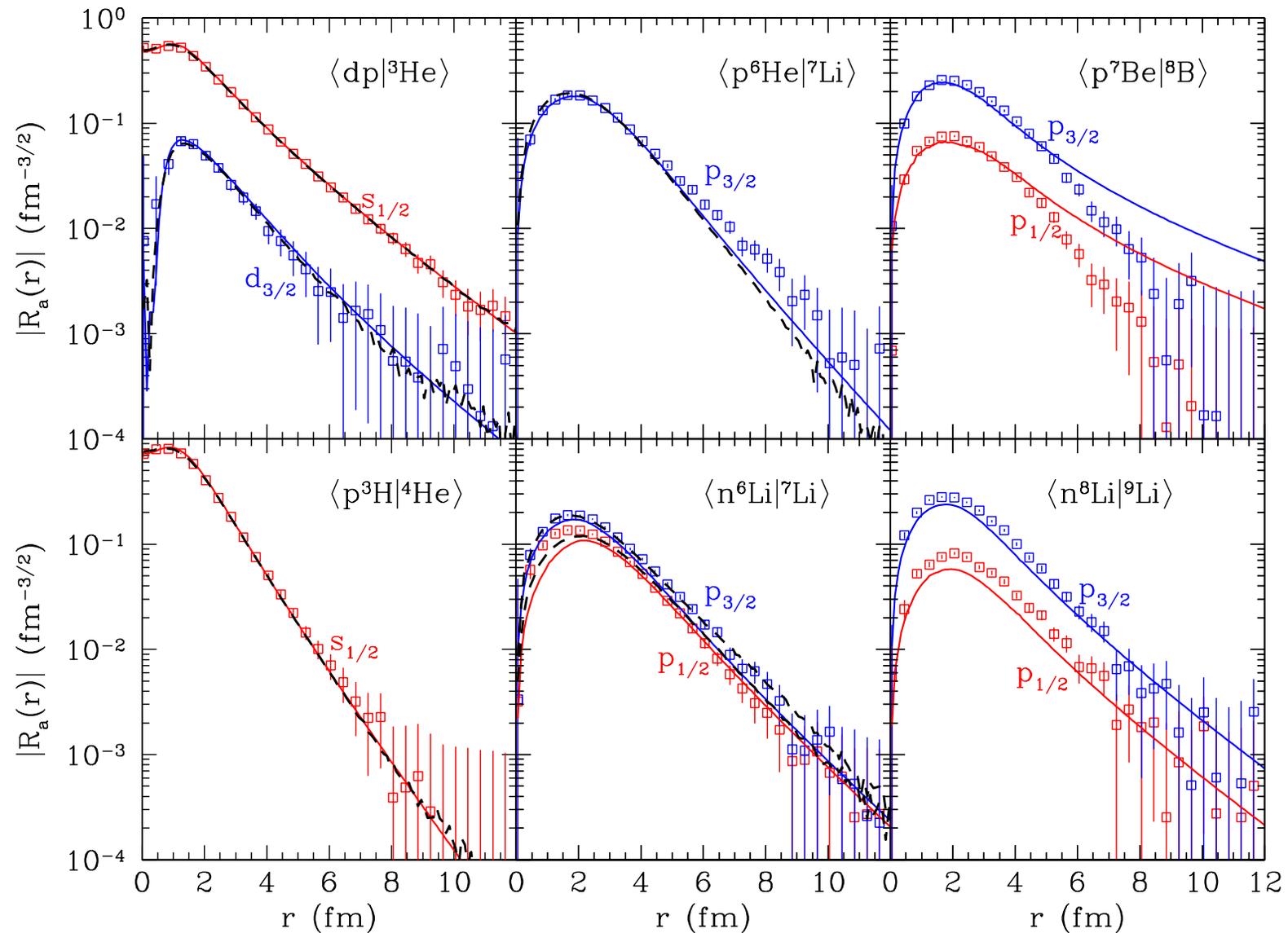
Asterisk: uncomputed channels

Dynamic range of 0.0005 to  $\lesssim 1.0$  MeV, not otherwise possible for QMC

Nollett, PRC in press, arXiv:1206.0046

## Overlaps at all radii: not just asymptotics

The integral relations contain more information about the potential than does the VMC wave function  $\rightarrow$  better overlaps



## Applications to “recent” experimental widths

- + agrees with experiment
- disagrees with experiment
- 0 can't tell because of wave function problems

State	Status	Expt ref.
${}^7\text{He}(\frac{3}{2}^-)$	+	
${}^7\text{He}(\frac{1}{2}^-)$	0	
${}^7\text{He}(\frac{5}{2}^-)$	0	
${}^8\text{B}(0^+)$	0	Mitchell 2010
${}^8\text{B}(2^+)$	–	Mitchell 2010
${}^9\text{Li}(\frac{5}{2}^-)$	+	Wuosmaa 2005
${}^9\text{Li}(\frac{3}{2}^-)$	+	Wuosmaa 2005
${}^9\text{Li}(\frac{7}{2}^-)$	+	Wuosmaa 2005
${}^9\text{He}$	?	Various

I find good support for Wuosmaa *et al.*  $J^\pi$  assignments in  ${}^9\text{Li}$

${}^9\text{He}$  is a mess:

I only compute  $\frac{1}{2}^-$  well, and it should have  $\Gamma \sim 1$  to 2 MeV

Very few claimed resonances can match that

## What next?

$\alpha$  (and other cluster) widths & overlaps once the code is more-generally written

Tests against scattering calculations to see whether I can get the AV18+UIX widths this way

GFMC and IL7 (better match to experimental thresholds)

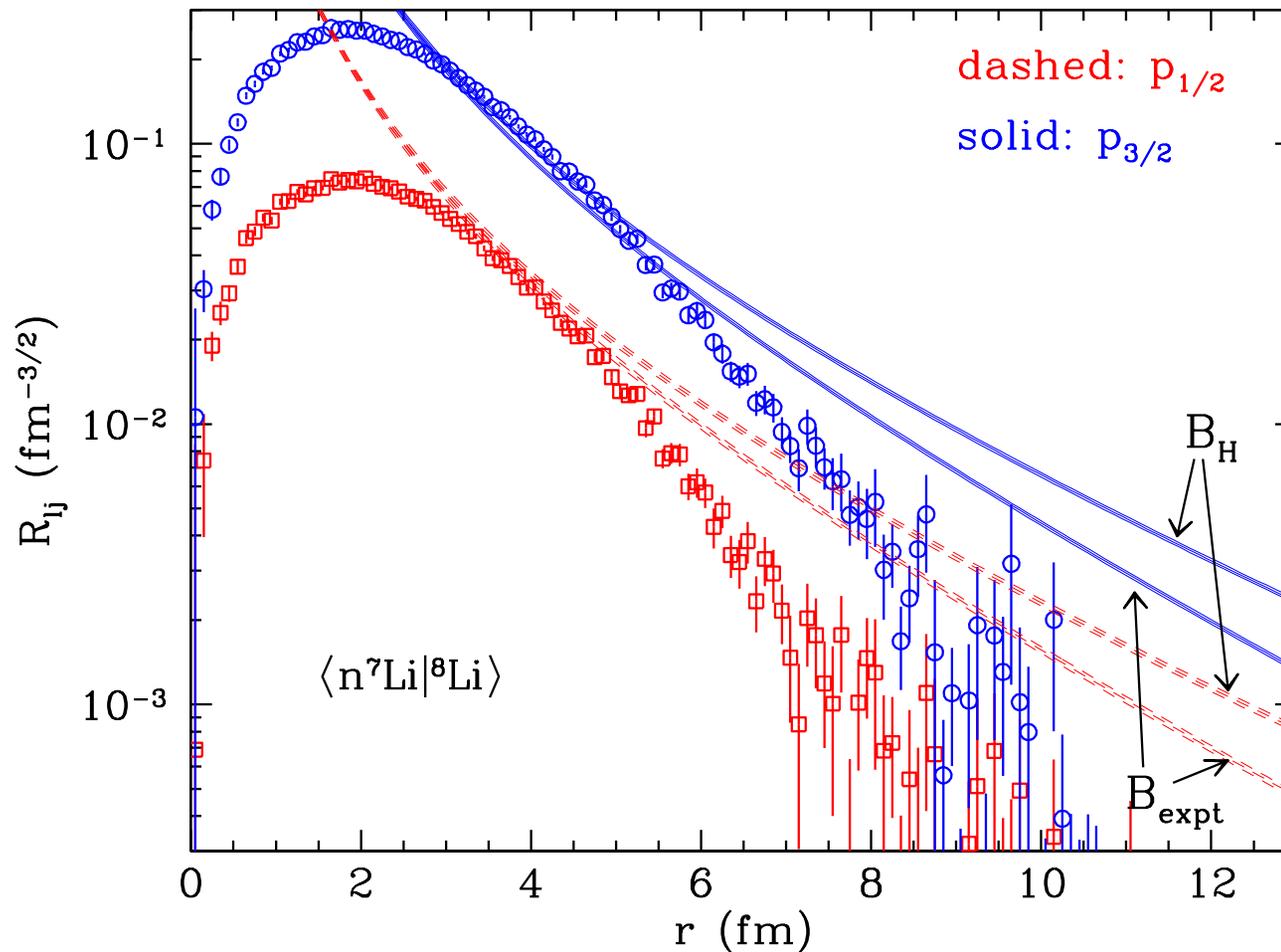
Similar things are being done as pseudobound approaches to scattering  $\delta(E)$  (Horiuchi et al., Kievsky et al., etc.) – some of that can be adapted

Coupled-channel problems will require some way of extracting surface amplitudes from GFMC, integrals are probably the way to do that

Energy resolutions below the 100 keV range are difficult for GFMC, so the integral approach will beat phase-shift mapping for really narrow states

BONUS MATERIAL

${}^8\text{Li} \rightarrow {}^7\text{Li} + n$  summarizes the whole project



ANC ( $\text{fm}^{-1}$ )	VMC: AV18+UIX binding	VMC: Lab binding	Experiment
$C_{p1/2}^2$	0.029(2)	0.048(3)	0.048(6)
$C_{p3/2}^2$	0.237(9)	0.382(14)	0.384(38)

# Testing out the integral relation for $\Gamma$

This has been a long time coming, paper in production now

State	Daughter	Experiment		From Exp energy	From AV18+UIX energy		Matches 90°?	$\zeta$
		$E$ (MeV)	$\Gamma$ (MeV)	$\Gamma_{VMC}$ (MeV)	$E_{UIX}$ (MeV)	$\Gamma_{VMC}$ (MeV)		
${}^5\text{He}(3/2^-)$	${}^4\text{He}(0^+)$	0.798	0.648 [50]	0.307(5)	1.39	0.684(11)	no	0.460
${}^5\text{He}(1/2^-)$	${}^4\text{He}(0^+)$	2.07	5.57 [50]	0.582(13)	2.4	0.711(15)	no	0.429
${}^7\text{He}(3/2^-)$	${}^6\text{He}(0^+)$	0.445	0.15(2)	0.114(4)	2.3	1.184(9)	yes	0.092
${}^7\text{He}(1/2^-)$	${}^6\text{He}(0^+)$	3.045	–	1.98(9)	2.91	1.87(8)	no	0.092
${}^7\text{He}(1/2^-)$	${}^6\text{He}(2^+)$	1.25	–	0.42(3)	1.11	0.36(2)	yes	0.067
${}^7\text{He}(1/2^-)$	sum	3.045	2.0(1.0)	2.40(12) <sup>a</sup>	2.91	2.22(11) <sup>a</sup>		
${}^7\text{He}(5/2^-)$	${}^6\text{He}(2^+)$	1.57	1.99(17)	1.31(10) <sup>a</sup>	1.87	1.66(13) <sup>a</sup>	no	0.165
${}^7\text{Li}(5/2^-)$	${}^6\text{Li}(1^+)$	0.204	0.0646	0.0483(17) <sup>a</sup>	1.55	0.92(3) <sup>a</sup>	yes	0.055
${}^7\text{Be}(5/2^-)$	${}^6\text{Li}(1^+)$	1.60	0.19(5)	0.426(14) <sup>a</sup>	2.5	1.00(3) <sup>a</sup>	yes	0.055
${}^8\text{B}(1^+)$	${}^7\text{Be}(3/2^-)$	0.632	–	0.0383(14)	1.47	0.346(12)	yes	0.001
${}^8\text{B}(1^+)$	${}^7\text{Be}(1/2^-)$	0.203	–	0.00105(6)	1.38	0.51(3)	yes	0.003
${}^8\text{B}(1^+)$	sum		0.0357(6)	0.0394(14)		0.86(3)	yes	
${}^8\text{Li}(3^+)$	${}^7\text{Li}(3/2^-)$	0.223	0.032(3)	0.0344(18)	2.5	1.12(6)	yes	0.007
${}^8\text{B}(3^+)$	${}^7\text{Be}(3/2^-)$	2.18	0.39(4)	0.38(2)	2.4	0.46(2)	yes	0.007
${}^8\text{B}(0^+)$	${}^7\text{Be}(3/2^-)$	[2.56]	–	[0.65(4)]	2.39	0.57(3)	no	0.005
${}^8\text{B}(0^+)$	${}^7\text{Be}(1/2^-)$	[2.24]	–	[1.23(6)]	2.30	1.29(7)	no	0.004
${}^8\text{Li}(0^+)$	${}^7\text{Li}(3/2^-)$	[0.97]	–	[0.37(2)]	0.94	0.389(15)	no	0.005
${}^8\text{Li}(0^+)$	${}^7\text{Li}(1/2^-)$	[0.62]	–	[0.516(18)]	0.62	0.72(2)	no	0.004
${}^8\text{Be}(1^+) T = 1^b$	${}^7\text{Li}(3/2^-)$	0.385	–	0.0089(3)	1.2	0.152(3)	yes	0.003
${}^8\text{Be}(1^+) T = 0^b$	${}^7\text{Li}(3/2^-)$	0.895	–	0.150(4)	0.5	0.0354(10)	yes	0.003
${}^8\text{Be}(1^+) \text{sum}^b$	${}^7\text{Li}(3/2^-)$		0.149(6)	0.159(4)		0.187(3)	yes	
${}^8\text{Be}(3^+) T = 1^b$	${}^7\text{Li}(3/2^-)$	1.81	–	0.166(8)	3.68	0.60(3)	yes	0.007
${}^8\text{Be}(3^+) T = 0^b$	${}^7\text{Li}(3/2^-)$	1.98	–	0.314(14)	2.33	0.43(2)	yes	0.003
${}^8\text{Be}(3^+) T = 1^b$	${}^7\text{Be}(3/2^-)$	0.170	–	0.0115(6)	2.09	0.44(2)	yes	0.007
${}^8\text{Be}(3^+) T = 0^b$	${}^7\text{Be}(3/2^-)$	0.335	–	0.050(2)	0.74	0.161(8)	yes	0.004
${}^8\text{Be}(3^+) \text{sum}^b$	sum		0.50(3)	0.542(16)		1.63(4)	yes	
${}^9\text{Li}(5/2^-)$	${}^8\text{Li}(2^+)$	0.232	0.10(3)	0.145(4)	0.97	1.17(3)	yes	0.003
${}^9\text{Li}(7/2^-)$	${}^8\text{Li}(2^+)$	2.366	–	0.0012(7)	3.64	0.0031(16)	no	0.045
${}^9\text{Li}(7/2^-)$	${}^8\text{Li}(3^+)$	0.111	–	0.0427(8)	0.23	0.126(3)	yes	0.006
${}^9\text{Li}(7/2^-)$	sum		0.04(2)	0.0439(11)		0.129(3)		
${}^9\text{Li}(3/2^-)$	${}^8\text{Li}(2^+)$	1.316	–	0.522(13)	1.51	0.631(17)	no	0.014
${}^9\text{Li}(3/2^-)$	${}^8\text{Li}(1^+)$	0.340	–	0.172(4)	0.50	0.302(8)	yes	0.006
${}^9\text{Li}(3/2^-)$	sum		0.6(1)	0.694(18)		0.932(19)		
${}^9\text{C}(1/2^-)$	${}^8\text{B}(2^+)$	0.918	0.10(2)	0.102(3)	1.54	0.428(11)	yes	0.006
${}^9\text{Be}(1/2^-)$	${}^8\text{Be}(0^+)$	1.110	0.86(9)	0.80(2)	4.37	4.89(12)	yes	0.0005
${}^9\text{B}(3/2^-)$	${}^8\text{Be}(0^+)$	0.185	0.00054(21)	0.00058(2)	1.9	0.92(2)	yes	0.0003
${}^9\text{Be}(7/2^-)$	${}^8\text{Be}(0^+)$	4.715	–	0.0082(4)	–	–	yes	0.005
${}^9\text{Be}(7/2^-)$	${}^8\text{Be}(2^+)$	1.685	–	0.40(2)	–	–	yes	0.003
${}^9\text{Be}(7/2^-)$	sum		1.2(2)	0.41(2) <sup>a</sup>	–	–	yes	
${}^9\text{B}(7/2^-)$	${}^8\text{Be}(2^+)$	4.13	2.0(2)	0.82(4) <sup>a</sup>	–	–	yes	0.003
${}^8\text{B}(2^+)$	${}^7\text{Be}(3/2^-)$	2.41	0.12(4)	0.425(15)	–	–	yes	0.004
${}^8\text{B}(2^+)$	${}^7\text{Be}(1/2^-)$	1.98	0.24(11)	0.039(2)	–	–	yes	0.010
${}^8\text{Li}(2^+)$	${}^7\text{Li}(3/2^-)$	[2.18]	–	[1.00(4)]	–	–	yes	0.004
${}^8\text{Li}(2^+)$	${}^7\text{Li}(1/2^-)$	[2.06]	–	[0.105(6)]	–	–	yes	0.010

## Widths and state identification: ${}^7\text{He}$ & ${}^9\text{Li}$

New theoretical information should be useful for  $J^\pi$  identification of states

The  ${}^7\text{He}$  ground state ( $\frac{3}{2}^-$ ) is not too bad:  $\Gamma = 114(4)$  keV vs.  $\Gamma = 125_{-15}^{+40}$  measured

Neither  ${}^7\text{He}(\frac{1}{2}^-)$  nor  ${}^7\text{He}(\frac{5}{2}^-)$  overlaps look like  $90^\circ$  phase shift (both are broad);  $\frac{1}{2}^-$  width isn't bad

Computed  ${}^9\text{Li}$  widths support  $J^\pi$  assignments of Wuosmaa *et al.* 2005:

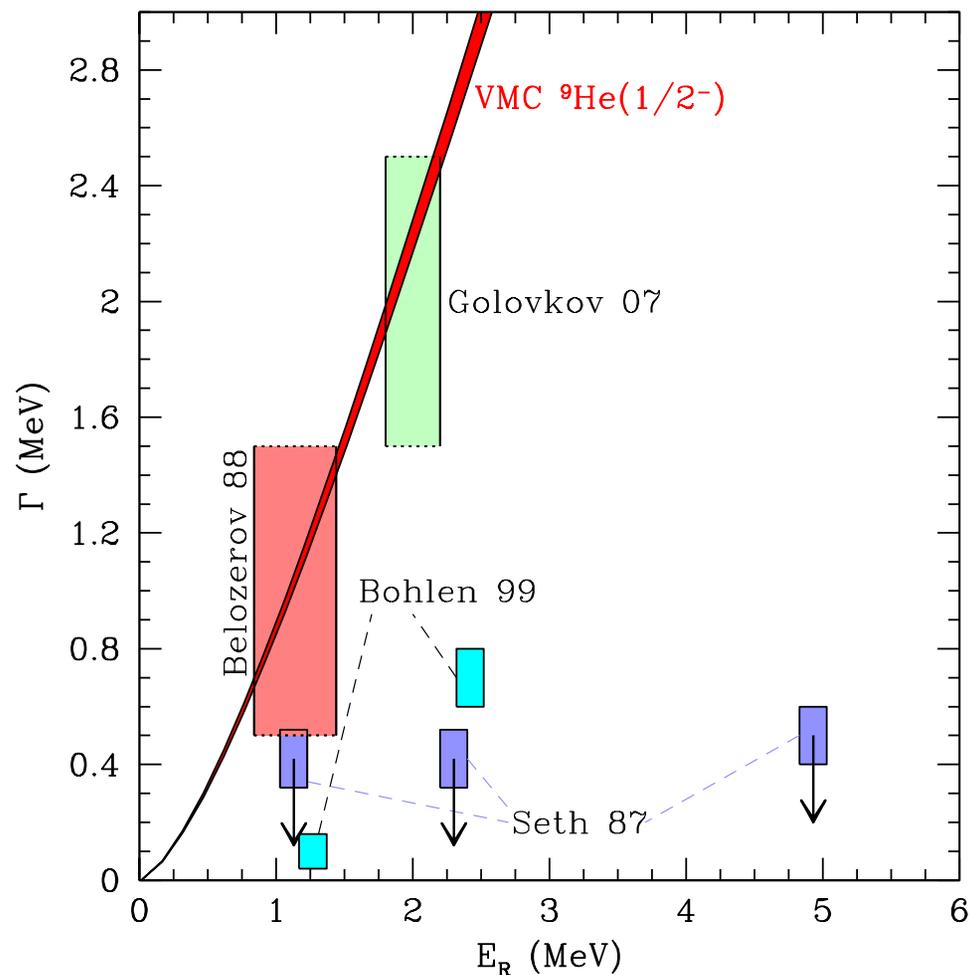
$3/2^-$ ,  $1/2^-$ ,  $5/2^-$ ,  $3/2^-$ ,  $7/2^-$

## Widths and state identification: ${}^9\text{He}$

Broad  $\frac{1}{2}^-$  matches width claimed at Dubna (but not elsewhere)

I find  $< 5$  keV width for  $\frac{3}{2}^-$ , but direct overlap is inconsistent with  $90^\circ$

Did not consider unbound decay products (so no decays through  ${}^8\text{He}(2^+)$ )



There should be even-parity intruders, but those VMC aren't well developed, and  ${}^8\text{He}(2^+)$  should be important

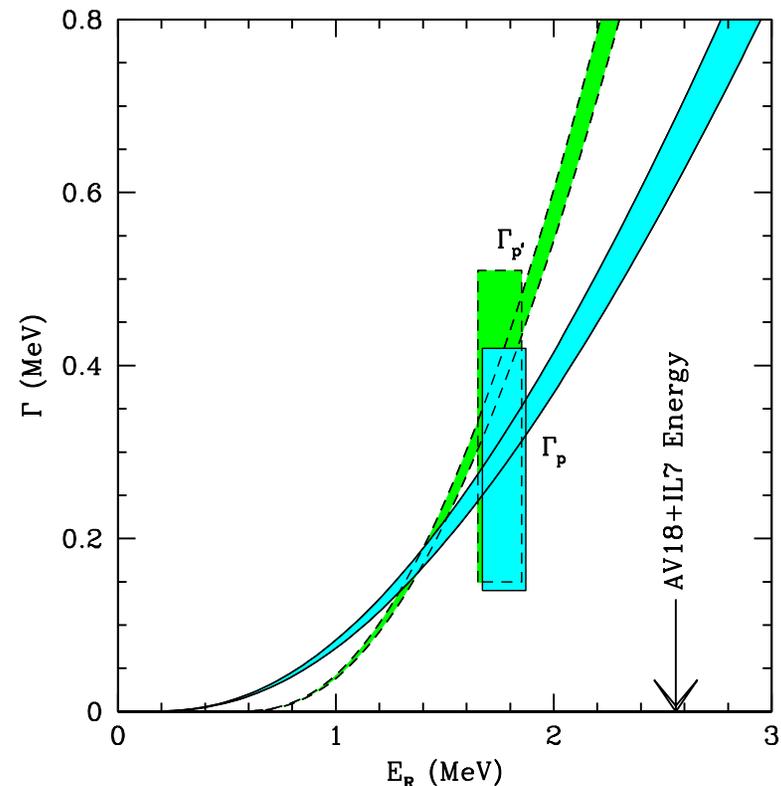
## Widths and state identification: $^8\text{B}$

Mitchell *et al.* 2010 claim new broad  $0^+$  &  $2^+$  states in  $^8\text{B}$  (at low significance)

$0^+$  width calculations look unreliable –  
90° test failed

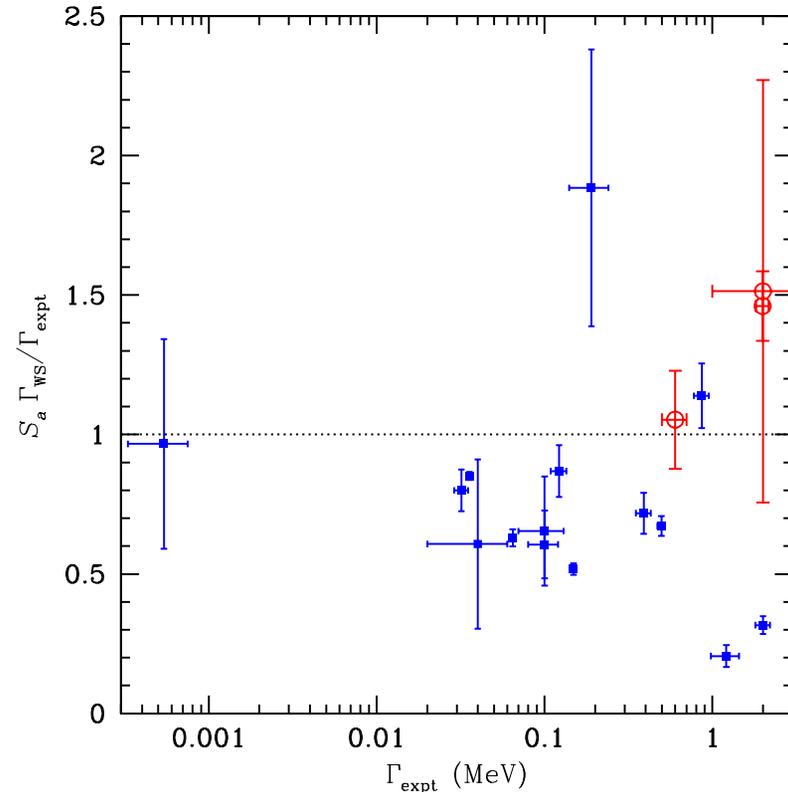
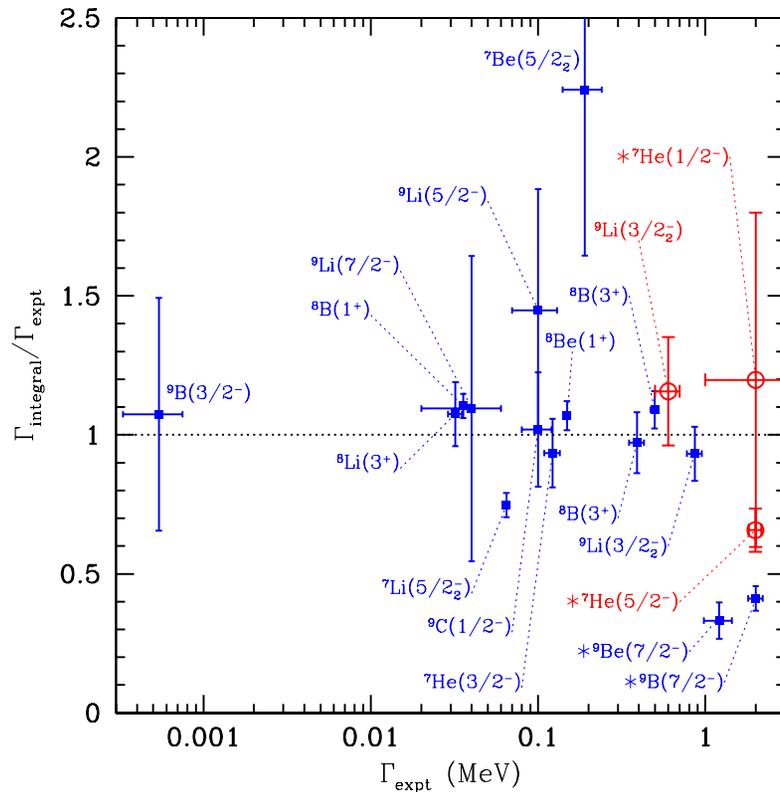
I can assume a range of  $E$  in the width  
and see what  $\Gamma$  corresponds

Widths to  $^7\text{Be}$  &  $^7\text{Be}^*$  computed  
separately



VMC  $2^+$  states are compatible with 90°, but I don't reproduce Mitchell widths  
of  $^8\text{B}(2_2^+)$  state

$$\Gamma \neq S_{lj} \Gamma_{s.p.}$$



blue: consistent with 90° via P-S

red: not consistent

For narrow states without open  $\alpha$  channels, the integral relation beats width estimates based on spectroscopic factors

Mean of vertical axis, states where all channels counted & VMC wave function “looks resonant:”  $1.06 \pm 0.07$  integral,  $0.75 \pm 0.15$  Woods-Saxon ( $\chi^2_{\nu} = 1.5$  vs. 34)

## From the beginning: Why/what is *ab initio* nuclear theory?

We seek the most fundamental description of the nucleus possible

QCD is at the moment too fundamental to be useful, so for now, *ab initio* theory means describing nuclei in terms of the NN interaction in vacuum

We want to understand how nuclear properties arise from properties of the interactions between nucleons

We want to constrain interactions for use *e.g.* in nuclear matter & neutron-star matter problems

We want to be able to predict nuclear cross sections for astrophysics

In the last 20 years, some important pieces of the *ab initio* puzzle have fallen into place

### Realistic potentials

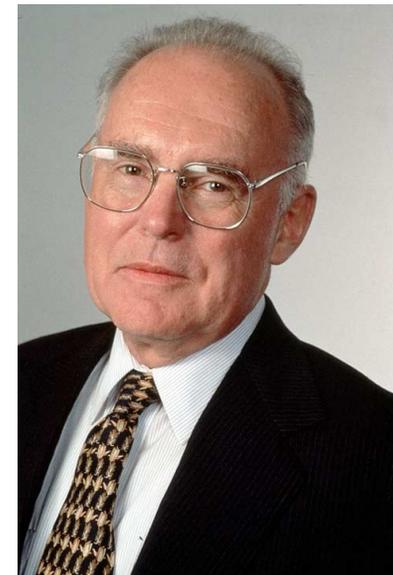
Several representations of the NN interaction were fitted to Nijmegen database of 4300 NN scattering data with  $\chi^2_{\nu} \simeq 1.0$  (CD Bonn, Nijmegen I & II, Argonne  $v_{18}$ , N<sup>3</sup>LO)

### Efficacious many-body methods

Correlated hyperspherical harmonic, Faddeev methods (for  $A \leq 4$ ); quantum Monte Carlo, no-core shell model, coupled-cluster, molecular dynamics methods (for  $A > 4$ )

### Expanding computer power

Moore's law provided the opportunity, but does not solve all problems



## The Hamiltonian

We work with the **Argonne  $v_{18}$**  nucleon-nucleon potential  
(18 operator terms, full EM, charge symmetry  
breaking,  $\chi^2_{\nu} = 1.09$ )



Three-nucleon interaction: Urbana IX fitted to  $^3\text{H}$  binding, saturation density  
Illinois-x fitted to 23 bound & narrow levels



This interaction is fed to variational Monte Carlo (VMC) and then Green's function  
Monte Carlo (GFMC) computational methods

## What we actually do, part I: Interactions

We work with the Argonne  $v_{18}$  nucleon-nucleon potential

It's one of the realistic potentials mentioned before



- fits all pp & np data to 350 MeV in Nijmegen 1993 phase shift analysis with  $\chi^2_{\nu} = 1.09$ , also deuteron properties
- 18 operator terms ( $\mathbf{L} \cdot \mathbf{S}$ ,  $\sigma \cdot \sigma$ , tensor, scalar...),  $\sim 40$  parameters **fitted once in 1995**
- local interaction, strong repulsive core, strong tensor interaction and  $\pi$  exchange at longer range
- full complication of EM interaction (mag. moment, vacuum polarization...), charge symmetry breaking, charge dependence

## What we actually do, part I: Interactions

In  $A \geq 3$  systems, there is an important **3-nucleon interaction** that provides a large fraction of the binding energy & spin-orbit splitting

Physically, this arises from lack of explicit  $\pi$  and  $\Delta$  d.o.f. in the wave function and is tangled with off-shell behavior of NN interaction

We use (mostly) the **Illinois 7 (IL7)** NNN interaction:

- 4 terms, spatial/spin/isospin dependence fixed by 2- & 3-pion exchange
- only **4 adjusted parameters** (strengths of those terms)
- fixed by fit to 23 bound and narrow levels at  $A \leq 8$
- RMS deviation of 700 keV from 60 experimental states in  $A \leq 10$



## Quantum Monte Carlo, part II: Methods

We want to find nuclear energies and wave functions from the interaction

$$\hat{H}\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

With one equation for each spin/isospin channel, this is some **270,000 coupled channels in 33 variables** for the case of  $^{12}\text{C}$

I use two methods as successive approximations:

- Variational Monte Carlo (VMC)
- Green's function Monte Carlo (GFMC)

Instead of a spatial basis, QMC methods operate on samples of the wave function at discrete points in the  $3A$ -dimensional configuration space

## What we actually do, part II: Methods

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## What we actually do, part II: Methods

Variational Monte Carlo (VMC) is based on a **sophisticated guess wave function** reflecting **pairwise interactions of nucleons**:

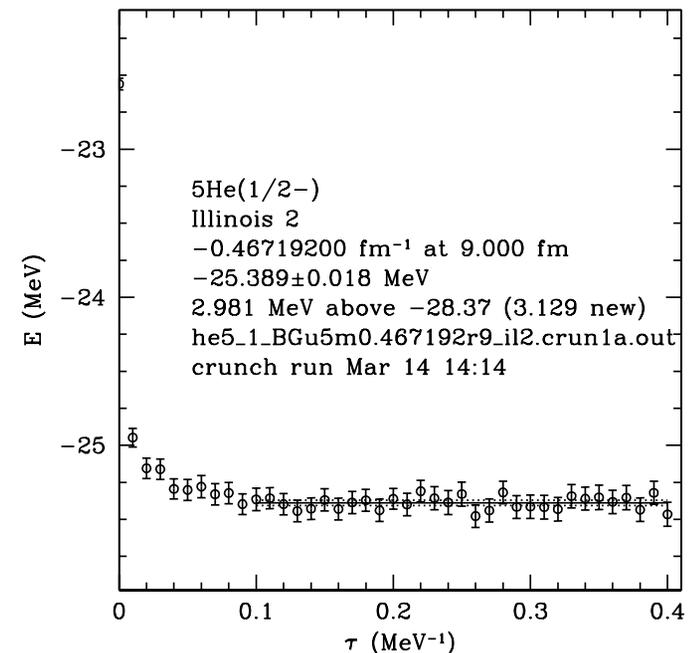
$$\Psi_T = [\text{3-body operator functions}] \times [\text{2-body operator functions}] \\ \times [\text{scalar functions}] \times [\text{shell-model-like orbital/spin/isospin structure}]$$

We evaluate  $E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle}$ , and set adjustable parameters by hand to **minimize  $E_T$**

Green's function Monte Carlo (GFMC) projects the true ground state out of the VMC wave function

$$\Psi(\tau) = \exp \left[ - \left( H - \tilde{E} \right) \tau \right] \Psi_T$$

As  $\tau \rightarrow \infty$ ,  $\Psi(\tau)$  approaches the ground state



## Quantum Monte Carlo results

The variational Monte Carlo (VMC) method give a first approximation, then Green's function Monte Carlo (GFMC) projects out the solution

Energies in 60 states up to  $A = 10$  are reproduced to better than 700 keV RMS (with four fitted parameters of the three-nucleon force)

We have also successfully computed:

- RMS radii and quadrupole moments are computed, mostly with success
- $\beta$  and  $\gamma$  transition rates
- $\alpha + n$  scattering phase shifts
- pickup and stripping cross sections (via form factors & spectroscopic factors)
- $(e, e'p)$  cross sections

## ANC: What is it good for?

Clearest case is low-energy direct capture,  $X + Y \longrightarrow Z + \gamma$

At  $E$  well below the Coulomb barrier, the initial-state wave function has very small amplitude in the nuclear interior (has to tunnel)

Matrix element then comes ( $\gtrsim 95\%$ ) from the asymptotic region so  $\sigma \propto C_{lj}^2$

Illustrative cases are  ${}^7\text{Be}(p, \gamma){}^8\text{B}$  &  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  in the sun (20 keV)

Approaches through the ANC have now been applied to  ${}^7\text{Be}(p, \gamma){}^8\text{B}$ ,  $d(\alpha, \gamma){}^6\text{Li}$ ,  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ ,  ${}^8\text{Be}(p, \gamma){}^9\text{C}$ ,  ${}^8\text{Li}(n, \gamma){}^9\text{Li}$ ,  ${}^{14}\text{N}(p, \gamma){}^{15}\text{O}$ ,  ${}^{12}\text{N}(p, \gamma){}^{13}\text{O}$ ,  ${}^{17}\text{F}(p, \gamma){}^{18}\text{Ne}$ , etc.

## ANCs in transfer reactions

Some ANCs can in principle be extracted from analytic continuation of scattering data (1970s)

Most ANC determinations for captures came from transfer, knockout, or breakup reactions

These are really special cases of spectroscopic-factor experiments

Such experiments are meant to probe the cluster overlap function

$$R_{lj}^{J_{A-1}J_A}(r) \equiv \int \mathcal{A} \left[ \Psi_{A-1}^{J_{A-1}} [\chi Y_l(\hat{\mathbf{r}})]_j \right]_{J_A}^\dagger \frac{\delta(r - r_{cc})}{r^2} \Psi_A^{J_A} d\mathbf{R}$$

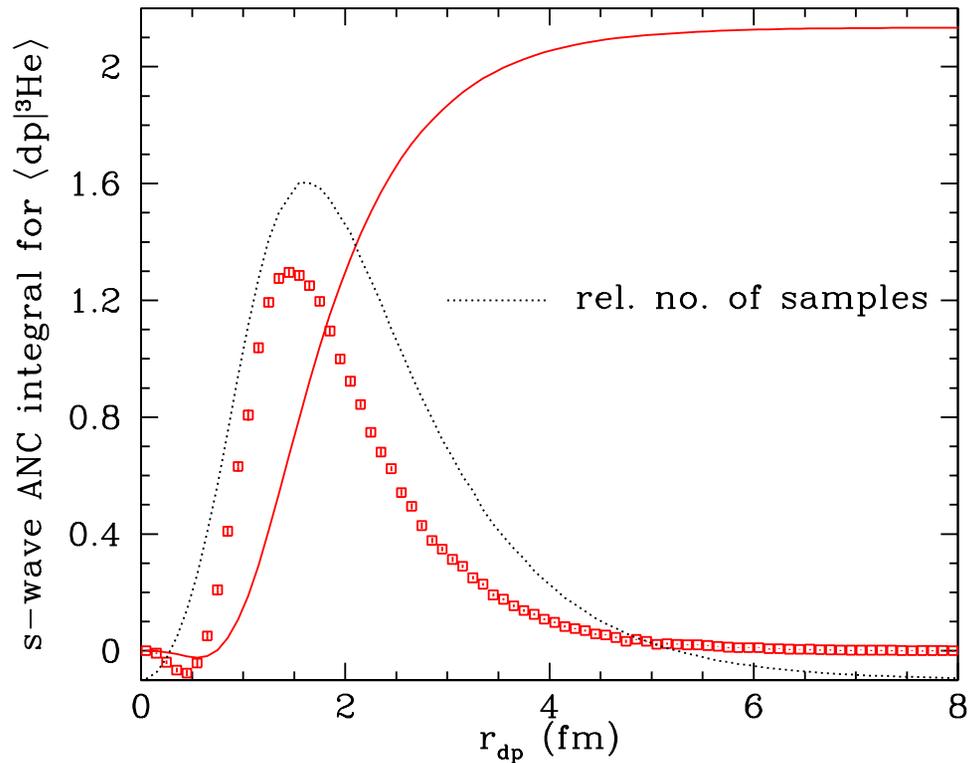
and particularly

$$S_{lj} \equiv \int R_{lj}^2(r) r^2 dr$$

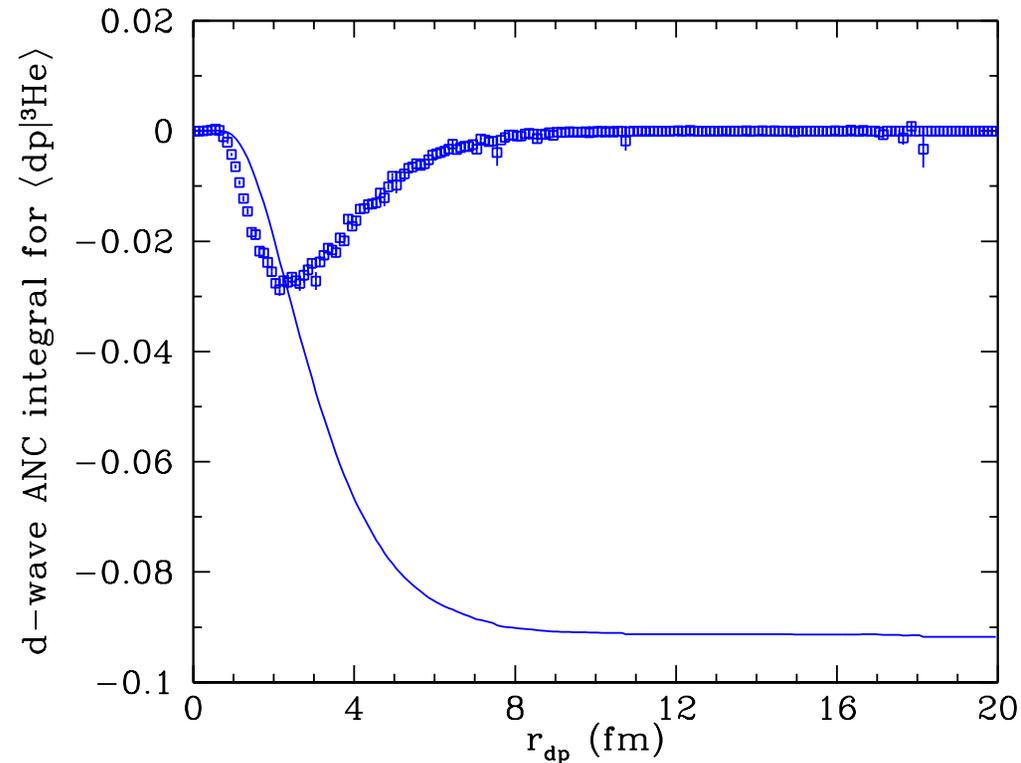
$R_{lj}$  and  $S_{lj}$  are easily defined and computed in a shell model and used in DWBA or other reaction theory

## ANCs: ${}^3\text{He} \rightarrow dp$

### s-wave ANC integrand & integral



### d-wave ANC integrand & integral



Points are Monte-Carlo sampled integrand; solid curves are cumulative integrals

For  ${}^3\text{He} \rightarrow dp$ , we have  $C_s^{dp} = 2.131(8) \text{ fm}^{-1/2}$ ,  $C_d^{dp} = -0.0885(7) \text{ fm}^{-1/2}$

$C_d^{dp}$  converges just where sampling gets sparse in the explicit overlap

## Application to the VMC wave functions

I've implemented the integral approach to the ANC within the VMC code, building on Wiringa's spectroscopic factor routines

I've applied the integral method to Wiringa's latest Argonne  $v_{18}$  + Urbana IX (AV18+UIX) wave functions for  $A \leq 9$  in almost every combination of particle stable  $A$ - and  $(A - 1)$ -body states

I have to choose a separation energy, either experimental or AV18+UIX, in evaluating each integral

It quickly became apparent that results match experiment only when the experimental separation energy is used

(Retrospective no-brainer: otherwise we're comparing against different functions)

## Why is any of this useful?

$$C_{lj} = \frac{2\mu}{k\hbar^2 w} \mathcal{A} \int \frac{M_{-\eta l + \frac{1}{2}}(2kr_{cc})}{r_{cc}} \Psi_{A-1}^\dagger \chi^\dagger Y_{lm}^\dagger(\hat{\mathbf{r}}_{cc}) (U_{\text{rel}} - V_C) \Psi_A d\mathbf{R}$$

The power of this approach lies in the factor  $(U_{\text{rel}} - V_C)$

It contains the potential, but only terms linking the core to the last particle:

$$U_{\text{rel}} = \sum_{i < A} v_{iA} + \sum_{i < j < A} V_{ijA}$$

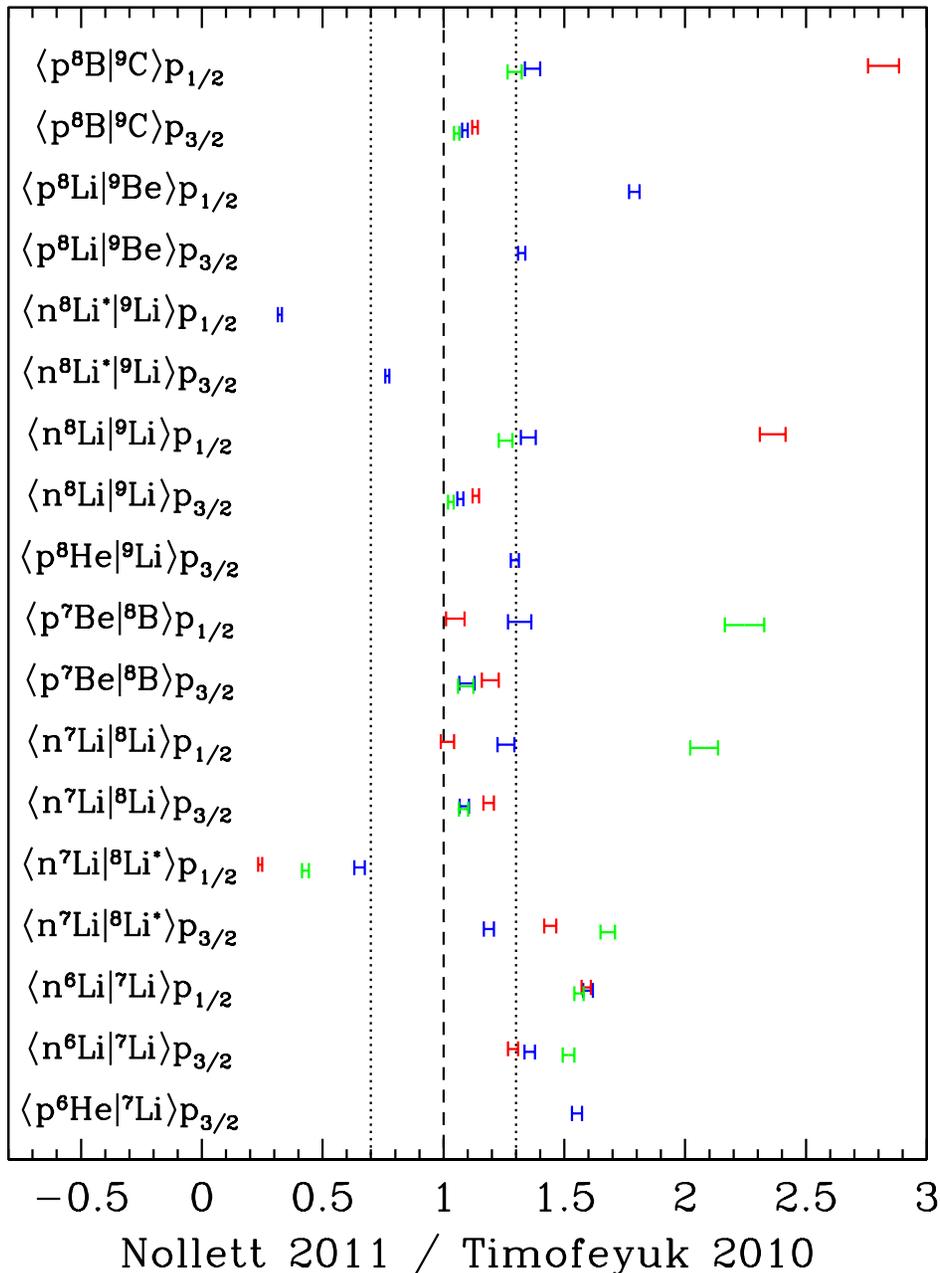
At large separation of the last nucleon,  $U_{\text{rel}} \rightarrow V_C$ , so  $U_{\text{rel}} - V_C \rightarrow 0$

Integrand goes to zero at  $r_{cc} \sim 7$  fm with AV18+UIX

QMC methods are good at integration over the wave function interior, bad at the exterior

Closely related to Lippman-Schwinger equation (and to Pinkston-Satchler or Kawai-Yazaki overlaps); used by Mukhamedzhanov & Timofeyuk since  $\sim 1990$

## Comparison with what came before



Timofeyuk has pursued a “hybrid” approach to the ANC integral for a long time

Wave functions come from p-shell model, integral from M3YE potential

Uncertainties have been hard to estimate

Colors denote shell model used in Timofeyuk 2010

Millener Boyarkina CK816

Attempts to derive ratios of isobaric-analogue ANCs from those calculations don't seem to hold up

## Heights and widths



“The other day I was walking my dog around my building, on the ledge. Some people are afraid of heights. I’m afraid of widths.”

– Steven Wright

We have *ab initio* energies for many narrow unbound levels (computed as bound)

Figuring out how to get widths has been difficult

There is an obvious but laborious way – explicit calculation of phase shifts at many energies, extraction of pole (has been done for  $^5\text{He}$  states)

Other paths have not panned out (e.g. “decay” rate in GFMC)

## Widths as ANC's

The relation

$$\psi(r \rightarrow \infty) = C_{lj} \phi_1 \phi_2 G_l(\eta, kr) / r$$

for resonant states is mathematically almost the same as

$$\psi(r \rightarrow \infty) = C_{lj} \phi_1 \phi_2 W_{-\eta, l + \frac{1}{2}}(2kr) / r$$

for bound states

The integral method also applies to resonant states, except that now  $F_l$  appears in the integral instead of  $M_{-\eta, l + \frac{1}{2}}$

This is used as a mathematical tool to get the asymptotics right in simpler  $\alpha$  and  $p$  decay models (e.g. Åberg et al. (1997) proton emitters, Russian literature on  $\alpha$  decay, etc.)

## Widths as ANCs

Hand-waving description of widths as ANCs:

An unbound wave function at large radius looks like

$$\psi(r \rightarrow \infty) \propto [F_l(kr) \cos \delta + G_l(kr) \sin \delta] / r$$

so that at resonance ( $\delta = 90^\circ$ ; as our pseudobound states should have)

$$\psi(r \rightarrow \infty) = C_{lj} \phi_1 \phi_2 G_l(kr) / r$$

The flux per unit time through the surface is  $|C_{lj}|^2 v = \frac{\hbar k}{\mu} |C_{lj}|^2$ , so

$$\Gamma \simeq \frac{\hbar^2 k}{\mu} |C_{lj}|^2$$

This is be shown to be nearly exact in papers by Humblet (not by this reasoning)

Widths can be computed (to some approximation) just like bound-state ANCs

## Overlaps at all radii

The ANC/width integrals are special cases of the overlaps of Pinkston & Satchler (or Kawai & Yazaki):

$$R_{lj}(r) \propto \left[ \cos \delta_{lj} + \int_r^\infty \frac{G_l(kr_{cc})}{r_{cc}} \Psi_{A-1}^\dagger \chi^\dagger(U_{\text{rel}} - V_C) \Psi_A d\mathbf{R} \right] F_l(kr)/r \\ + \left[ \int_0^r \frac{F_l(kr_{cc})}{r_{cc}} \Psi_{A-1}^\dagger \chi^\dagger(U_{\text{rel}} - V_C) \Psi_A d\mathbf{R} \right] G_l(kr)/r$$

90° phase shift means no  $F_l$  component at  $r \rightarrow \infty$

If this  $R_{lj}$  with  $\cos \delta_{lj} = 0$  is a poor match to the directly-computed overlap at small  $r$ , then  $\delta \neq 90^\circ$  for that channel  $\rightarrow$  my assumptions are invalid

Cases that fail this test generally have small spectroscopic factors

## Overlaps at all radii

The ANC/width integrals are special cases of the overlaps of Pinkston & Satchler  
(or Kawai & Yazaki)

By considering integral relations away from  $r \rightarrow \infty$  limit, we get overlap functions  
at all  $r$

These can be used in reaction models

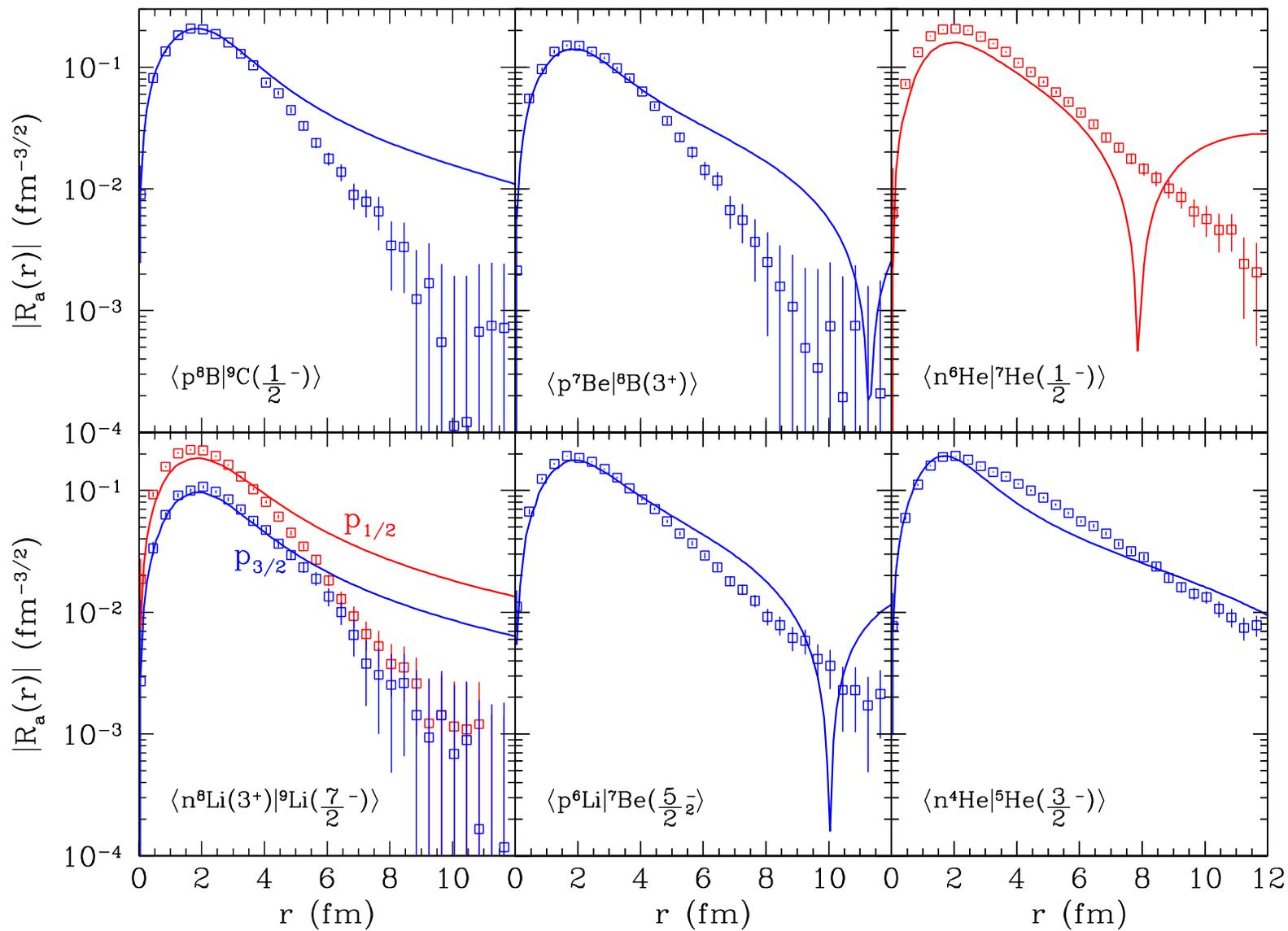
They also provide a test of whether unbound VMC wave functions really look  
like resonant states

# Overlaps at all radii

Good

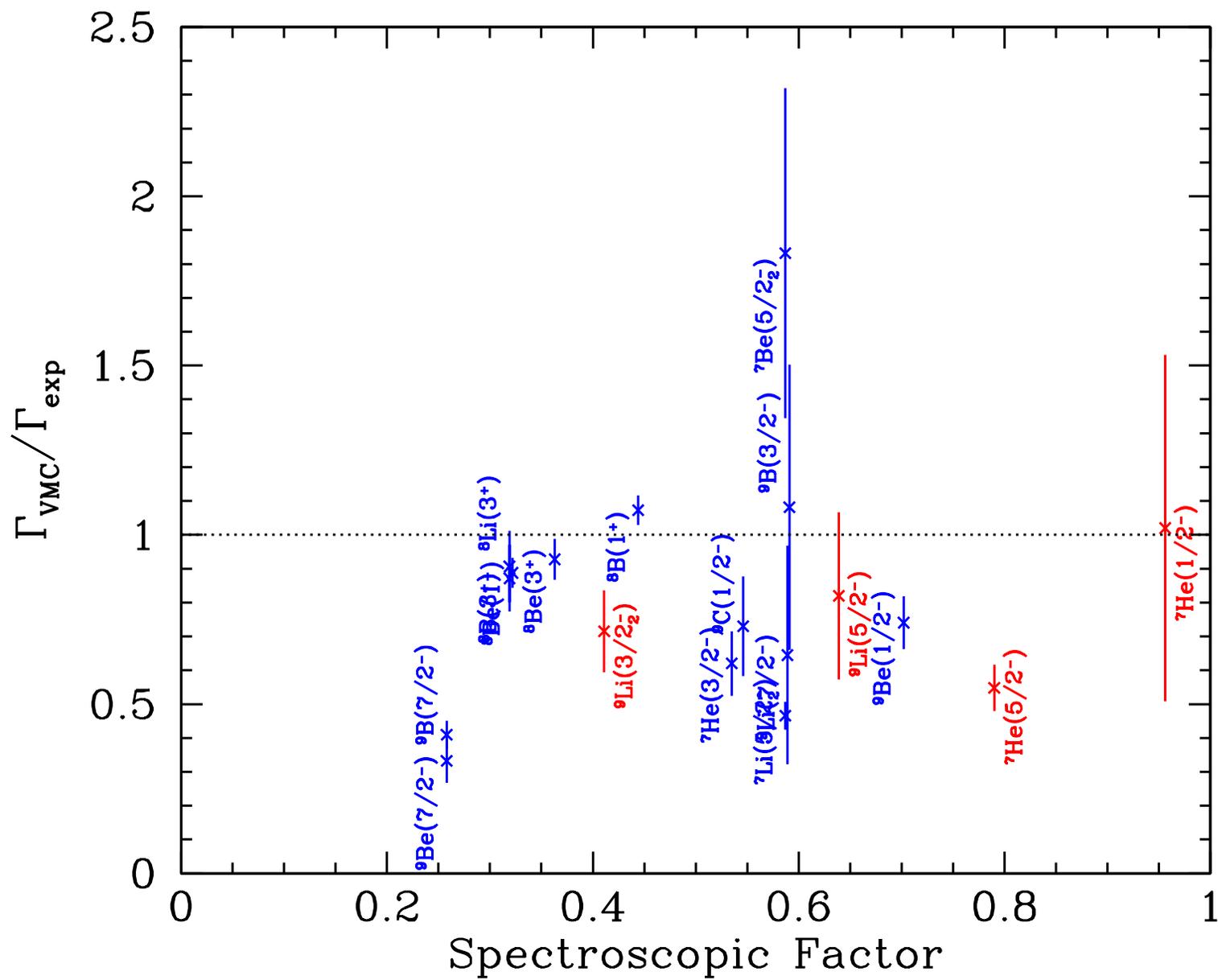
Good

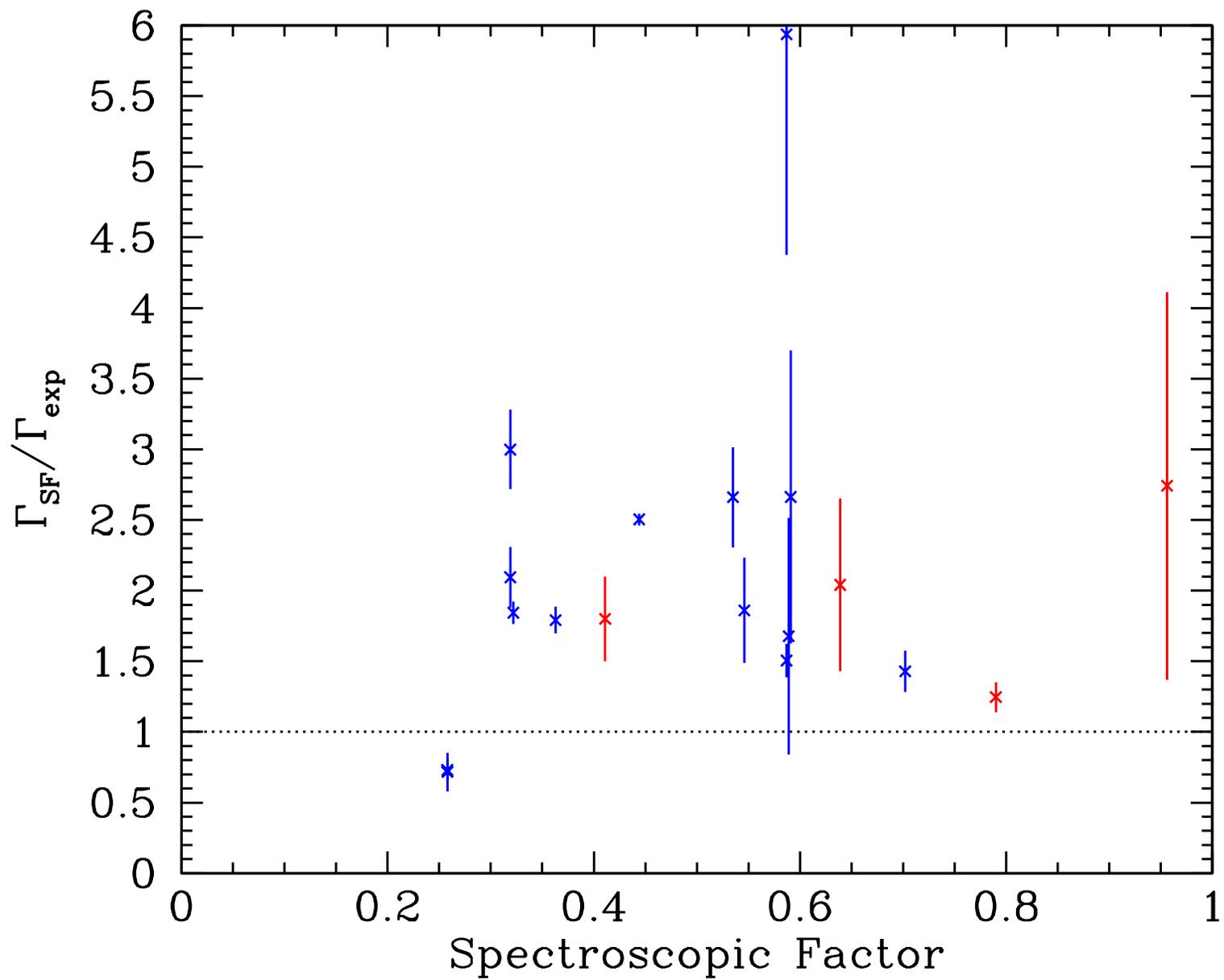
Bad



Points: Direct overlap

Curves: From integral relation





## Asymptotic Normalization Coefficient (ANC): Definition

Solutions  $\psi = u(r)/r$  to the one-body Schrödinger equation have known forms at large  $r$ :

$$-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} u(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} u(r) + U(r)u(r) + \frac{Z_1 Z_2 e^2}{r} u(r) = Eu(r)$$

$$-\frac{\hbar^2}{2\mu} \frac{d^2}{dr^2} u(r) + \frac{l(l+1)\hbar^2}{2\mu r^2} u(r) + \frac{Z_1 Z_2 e^2}{r} u(r) = Eu(r), \quad r \rightarrow \infty$$

$E < 0$  solutions to the second equation are the Whittaker functions  $W_{-\eta, l+1/2}(2kr)$

[For neutrons,  $Z_1 Z_2 = 0$  and  $W_{0, l+1/2}(2kr) = \sqrt{2kr/\pi} K_{l+1/2}(kr)$ , modified spherical Bessel function of the 3rd kind]

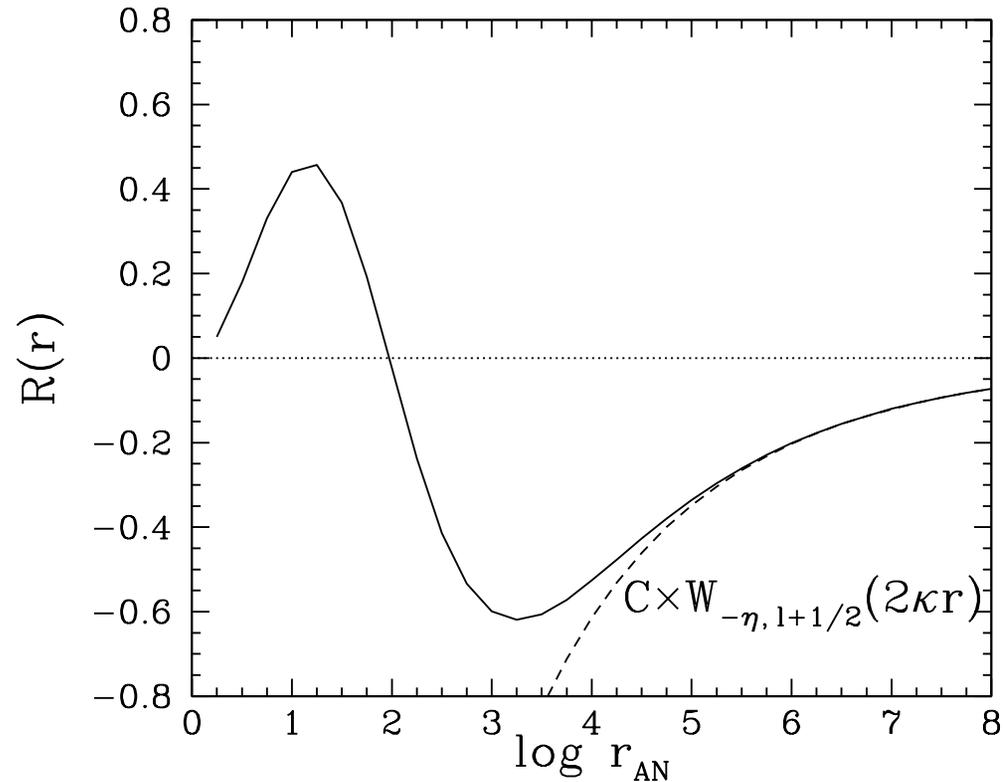
Solutions of the first equation must then satisfy the property that

$$\psi(r \rightarrow \infty) = CW_{-\eta, l+1/2}(2kr)/r,$$

where  $k = \sqrt{2\mu E}/\hbar$  and the ANC  $C$  depend on the short-range potential  $U(r)$

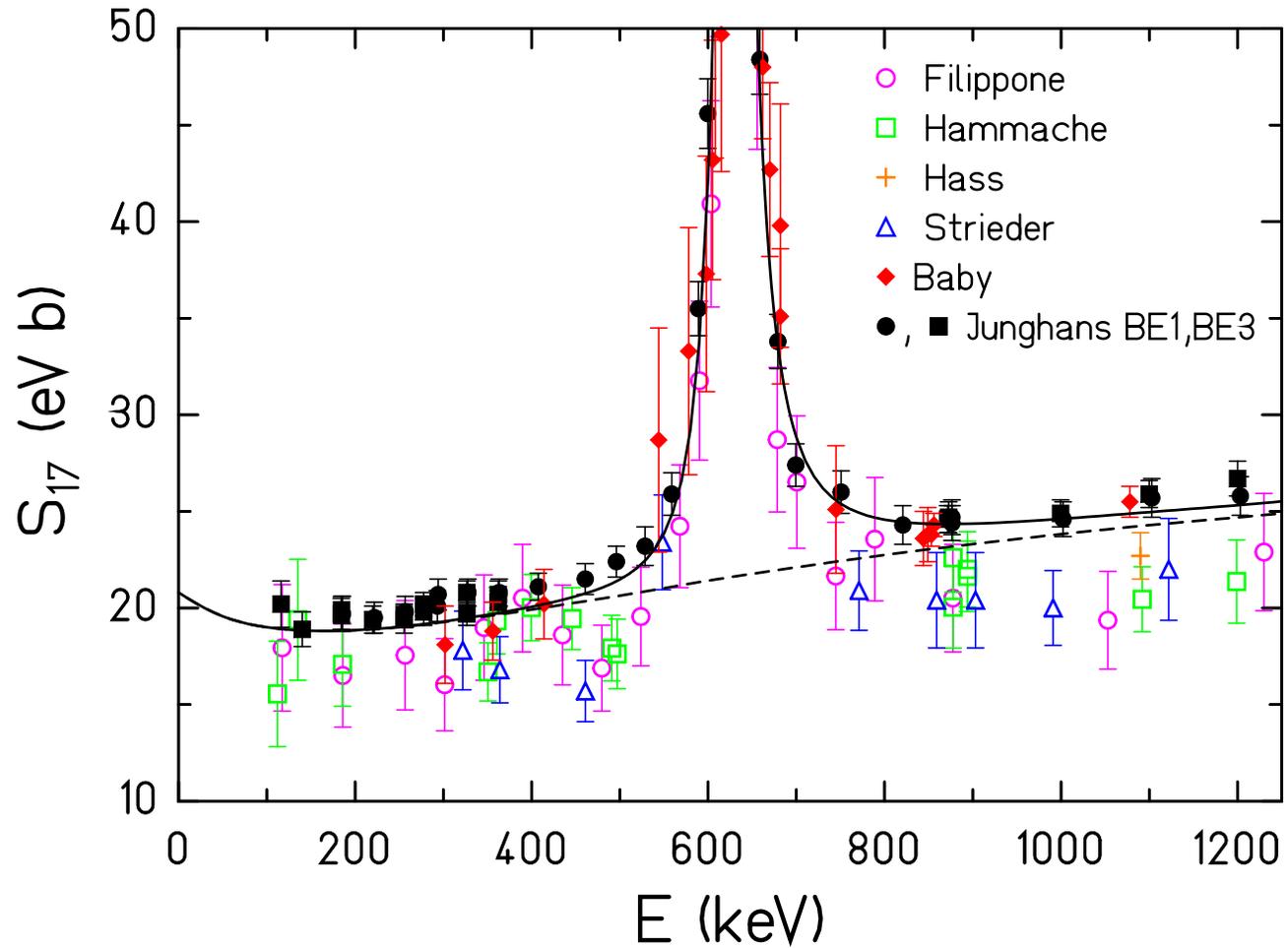
## ANC: example

So for example in a cluster model of  ${}^7\text{Li}$  as an  $\alpha$ -triton bound state, we have:



The short-range structure depends on details of the potential, but at long range the details reduce to  $C$  and  $E$

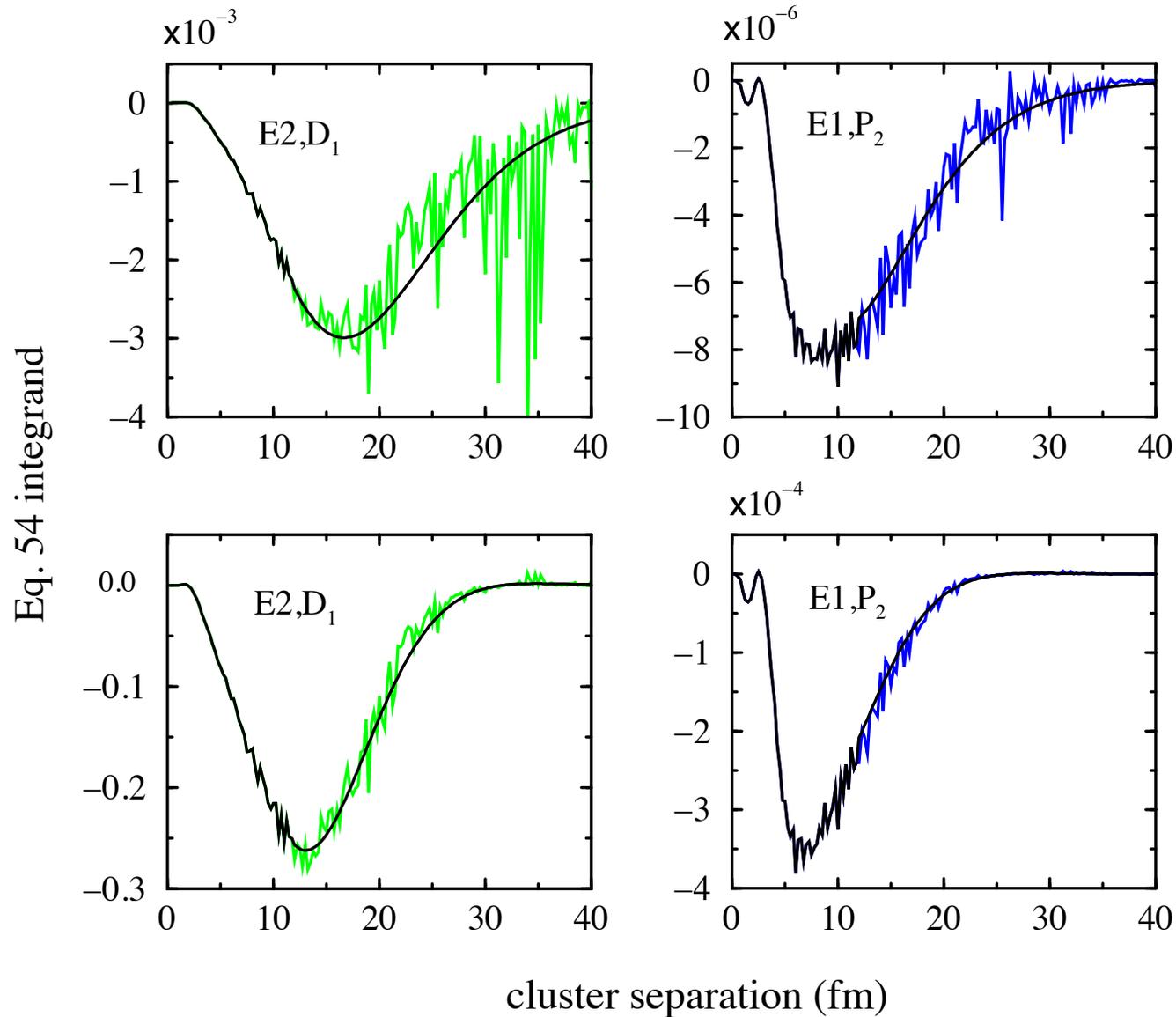
${}^7\text{Be}(p, \gamma){}^8\text{B}$  S-factor at low energy:



The upward turn at low  $E$  can be understood entirely from external capture

## External direct capture: $d(\alpha, \gamma)^6\text{Li}$

$d(\alpha, \gamma)^6\text{Li}$  matrix element density at 50 keV:



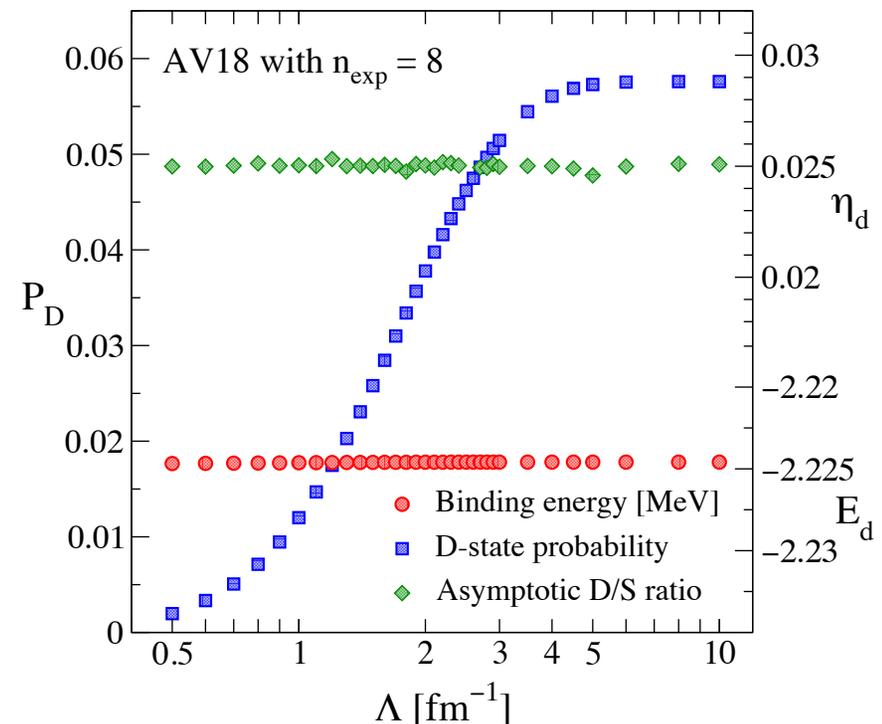
## ANC vs. spectroscopic factor

Some recent papers:

Mukhamedzhanov & Kadyrov 2010 show that  $S_{lj}$  is changed by unitary transformations of the potential, while  $C_{lj}$  is not

Jennings 2011 (“The Non-observability of Spectroscopic Factors”) demonstrates ambiguities that in principle let you dial  $S_{lj}$  to any desired number – but  $C_{lj}$  remains fixed

Bogner et al. 2007 shows how  $^2\text{H}$  d-state probability can be zeroed out by cutting out high-momentum components while preserving low- $E$  phase shifts (and thus ANCs)



## ANC vs. spectroscopic factor

The spectroscopic factor still has its place in an analysis where particularly  $R_{lj}$  is consistent between reaction model and structure model

$C_{lj}$  is more nearly an observable in the quantum mechanical sense (doesn't depend on representation, related to scattering amplitude)

$C_{lj}$  is more directly applicable to capture processes

$C_{lj}$  is easier to extract from a single analysis and compare with many models

$C_{lj}$  is generally tougher to get at, e.g. Jennings:

“Unfortunately, processes that are not strongly peaked are not uniquely determined by the asymptotic properties. Thus the bulk quantities we frequently want are not observables while the asymptotic properties are observables but do not contain the information necessary to fully specify the reaction of interest.”

## The ANC as observable (1980s)

In fact, there's a history going back to the 1970s

The triton d/s ANC ratio  $\eta_t$  emerged as an interesting quantity in Faddeev calculations

TABLE I. Theoretical and experimental determinations of  $\eta_t$ .

Method	$\eta_t$	Reference
Theory		
Correlation of $\eta_t$ and $E_t$	-0.0432(15)	[3]
Correlation of $\eta_t$ and $E_t$	-0.046(1)	[4]
Correlation of $\eta_t/\eta_d$ and $E_t$	-0.0430(12) <sup>a</sup>	[5]
Experiment		
Pole extrapolation, ${}^2\text{H}(\vec{d}, p){}^3\text{H}$	-0.048(7)	[6]
Pole extrapolation, ${}^2\text{H}(\vec{d}, p){}^3\text{H}$	-0.051(5)	[7]
Pole extrapolation, ${}^4\text{He}(\vec{d}, {}^3\text{He}){}^3\text{H}$	-0.050(6)	[8]
LEA DWBA analysis	-0.044(4)	[9]
DWBA analysis, ${}^{31}\text{P}(\vec{d}, t){}^{30}\text{P}$	-0.050(10)	[10]
Sub-Coulomb DWBA analysis	-0.043(4) <sup>b</sup>	[11]
This experiment	-0.0431(25)	—

<sup>a</sup> Using  $\eta_d = 0.0256(4)$  (Ref. [29]).

<sup>b</sup> Uncertainty from Ref. [16].

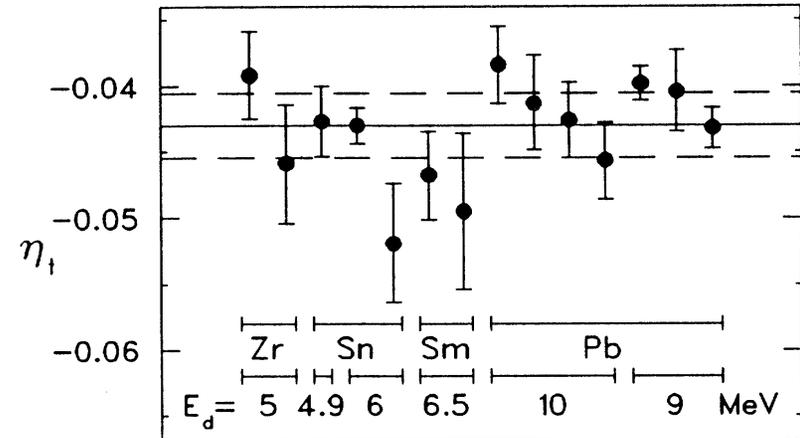


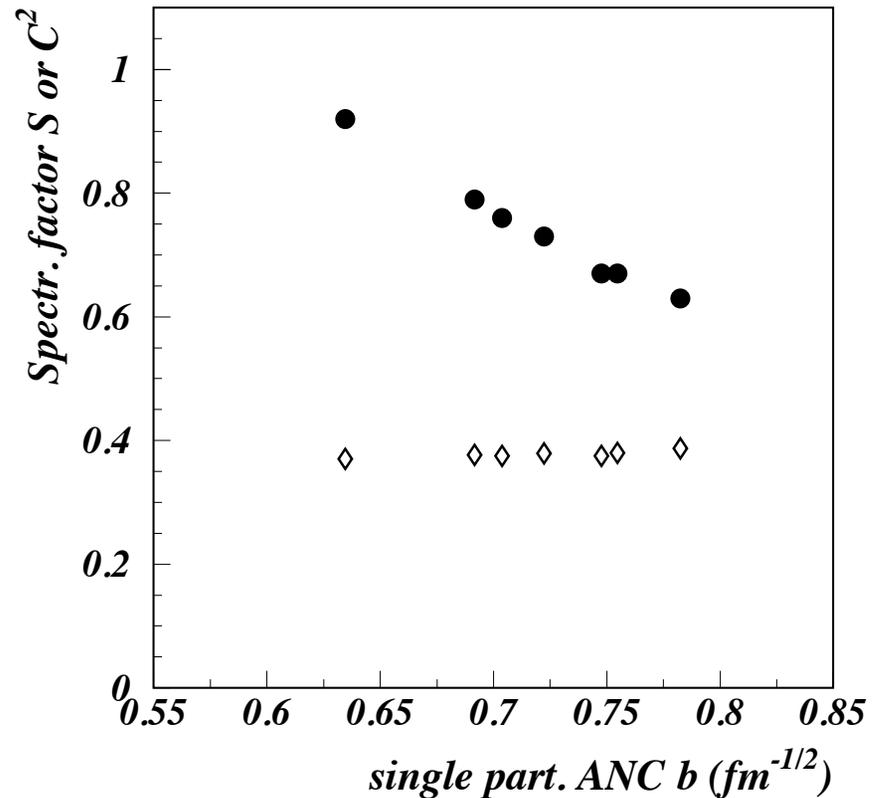
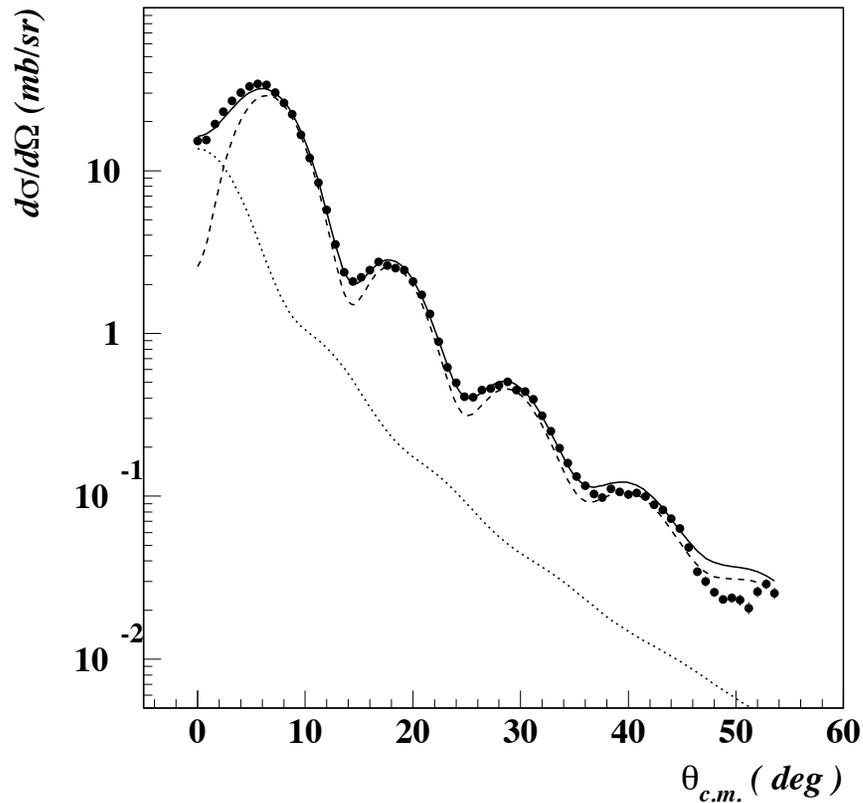
FIG. 6. Results for  $\eta_t$  for the 14 sub-Coulomb cases in Table II. The values are plotted in the order in which they are listed in the table. The error bars shown are the statistical

George & Knutson, PRC 48, 688 (1993)

Most precise approach ended up being tensor analyzing powers in sub-Coulomb transfers, e.g.  ${}^{208}\text{Pb}(\vec{d}, t)$  interference modeled with DWBA

## The ANC as observable (1990s)

The recent literature concentrates on transfer and knockout reactions, e.g.  ${}^8\text{Li}$   
ANCs from  ${}^{13}\text{C}({}^7\text{Li}, {}^8\text{Li}){}^{12}\text{C}$



Trache et al 2003

## Dispersion theory and related approaches to the ANC (1970s)

A bound state produces a pole in scattering amplitude at  $E < 0$

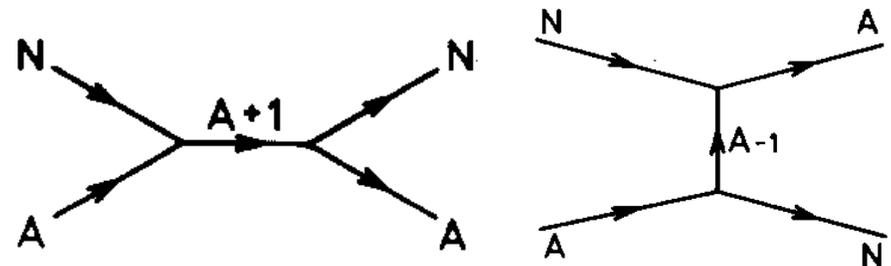
The ANC is proportional to the residue of that pole, the “vertex constant” or virtual width

Many authors extracted vertex constants from data, sometimes as pure extrapolation and sometimes using dispersion relations to subtract non-pole background

None of the numbers are quite experimental (few involved dedicated experiments)

Well-isolated poles not too far from threshold are necessary, and analytic continuation brings ambiguities

Need for isolated poles rules out many  $A > 4$  cases because excited states & exchange diagrams bring poles and branch cuts



## Relation between ANCs and observables

Clearest case is low-energy direct capture,  $X + Y \longrightarrow Z + \gamma$

At  $E$  well below the Coulomb barrier, the initial-state wave function has very small amplitude in the nuclear interior (has to tunnel), large  $r$  dominates matrix element,  $\sigma \propto C_{lj}^2$

Bound states produce negative-energy poles in the scattering amplitude  
→ ANCs  $\propto$  residues and can sometimes be extracted from analytically-continued scattering data (1970s)

Most ANC determinations (usually motivated by astrophysical capture) come from transfer, knockout, or breakup reactions

These are special cases of “spectroscopic factor” experiments, requiring demonstrated independence from small- $r$  contributions

## ANCs in transfer reactions

ANC or spectroscopic factor experiments are meant to probe the cluster overlap function

$$R_{lj}^{J_{A-1}J_A}(r) \equiv \int \mathcal{A} \left[ \Psi_{A-1}^{J_{A-1}} [\chi Y_l(\hat{\mathbf{r}})]_j \right] \dagger_{J_A} \frac{\delta(r - r_{cc})}{r^2} \Psi_A^{J_A} d\mathbf{R}$$

and particularly the spectroscopic factor

$$S_{lj} \equiv \int R_{lj}^2(r) r^2 dr$$

(though this is a questionable meeting point for theory and experiment; see recent papers by Mukhamedzhanov, Jennings, etc.)

Since  $R_{lj}(r \rightarrow \infty) = C_{lj} W_{-\eta, l + \frac{1}{2}}(2kr)/r$ , the ANC  $C_{lj}$  can in principle be isolated in data restricted to large impact parameter

Some of the usual limitations (e.g. optical potentials) apply just as well to  $C_{lj}$  as to  $S_{lj}$

Consistency of  $R_{lj}$  between reaction & structure theory is easier for  $C_{lj}$  than for  $S_{lj}$ , provided that you can prove peripherality

## Why quantum Monte Carlo ANCs require effort

GFMC requires all the work of variational Monte Carlo plus more, so for now I work with VMC wave functions:

$$\Psi_T = [\text{3-body operator functions}] \times [\text{2-body operator functions}] \\ \times [\text{scalar functions}] \times [\text{shell-model-like orbital/spin/isospin structure}]$$

Each piece contains variational parameters, found by minimizing energy as computed by Monte Carlo integration

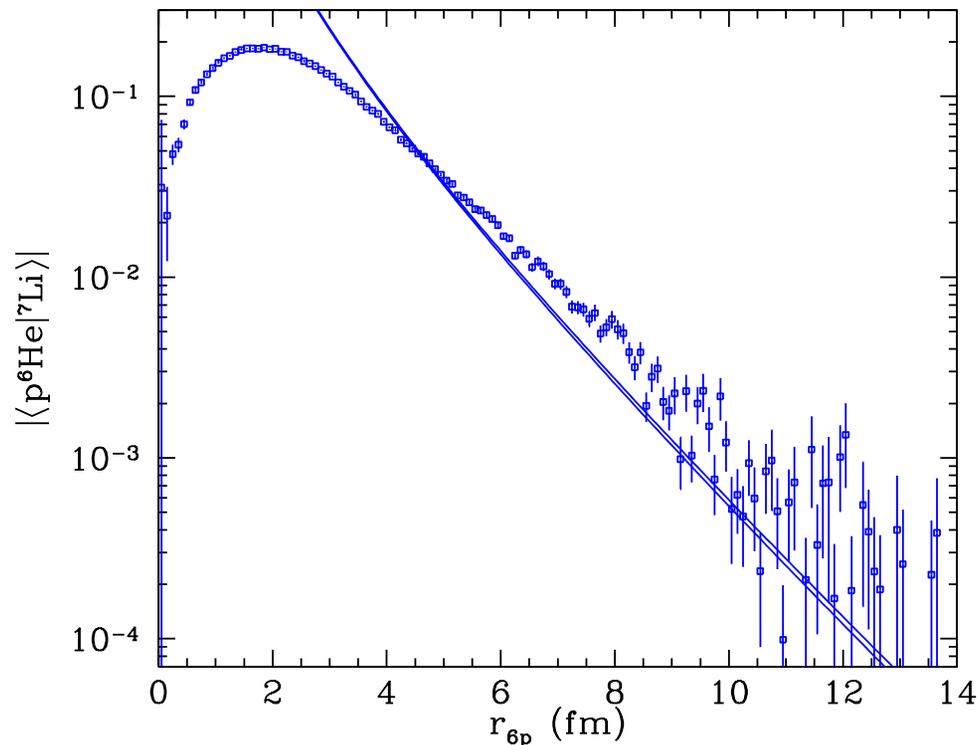
The VMC ansatz is very good and allows rather accurate calculations of energies and other observables (GFMC polishes VMC solutions down to the correct solution)

## Barriers to getting ANCs from quantum Monte Carlo calculations

The VMC wave functions account very well for short-range correlations but generally get the long-range asymptotics wrong

Correcting the long-range problems in a given clusterization channel without causing other problems is difficult (other channels get wrecked...)

$C_{lj} = rR_{lj}(r)/W_{-\eta,l+\frac{1}{2}}(2kr)$  doesn't work because long-range shapes are generally wrong



Points are  $R_{lj}$  from VMC

Overlap is a Monte Carlo integration

Curve is  $W_{-\eta,l+\frac{1}{2}}(2kr)/r$

Where do I match them?

Basis methods have the same problem

## Barriers to getting ANCs from quantum Monte Carlo calculations

Even if I have the correct solution for the potential, the potential may not match experimental separation energies

That again gives the wrong shape (not so bad for Illinois forces)

OK for comparison to other calculations; not good for predictions

In QMC methods, the  $R_{lj}$  overlap integral is evaluated by Monte Carlo integration over particle coordinates

Sampling based on simplified wave functions typically falls apart just where  $R_{lj}$  becomes asymptotic

Other sampling schemes are even worse

## Integral relation for the ANC

There is a better way than explicit overlaps, ideally suited to QMC methods

Consider the  $A$ -body wave function  $\Psi_A$  and its overlap with  $\Psi_{A-1}$  plus a final proton (separation energy  $B$ )

Write the Schrödinger equation as

$$(H - E) \Psi_A = 0$$

and expand  $H$  and  $E$  into parts internal to  $\Psi_{A-1}$  and parts involving the last particle

$$(H_{\text{int}} + T_{\text{rel}} + U_{\text{rel}} + V_C - V_C - E_{\text{int}} + B) \Psi_A = 0$$

Then

$$\begin{aligned} \Psi_A = & - [T_{\text{rel}} + V_C + B]^{-1} (U_{\text{rel}} - V_C) \Psi_A \\ & - [T_{\text{rel}} + V_C + B]^{-1} (H_{\text{int}} - E_{\text{int}}) \Psi_A \end{aligned}$$

The second line is zero since  $(H_{\text{int}} - E_{\text{int}}) \Psi_{A-1} = 0$

## Integral relation for the ANC

Rewriting the Green's function  $[T_{\text{rel}} + V_C + B]^{-1}$  in terms of special functions turns

$$\Psi_A = - [T_{\text{rel}} + V_C + B]^{-1} (U_{\text{rel}} - V_C) \Psi_A$$

into

$$\Psi_{A-1}^\dagger \chi^\dagger Y_{lm}^\dagger \Psi_A = \frac{2\mu}{k\hbar^2 w} \mathcal{A} \int \frac{M_{-\eta, l+\frac{1}{2}}(2kr_<) W_{-\eta, l+\frac{1}{2}}(2kr_>)}{r_< r_>} \times \Psi_{A-1}^\dagger \chi^\dagger Y_{lm}^\dagger(\hat{\mathbf{r}}_{cc}) (U_{\text{rel}} - V_C) \Psi_A d\mathbf{R}$$

so at large radius

$$C_{lj} = \frac{2\mu}{k\hbar^2 w} \mathcal{A} \int \frac{M_{-\eta, l+\frac{1}{2}}(2kr_{cc})}{r_{cc}} \Psi_{A-1}^\dagger \chi^\dagger Y_{lm}^\dagger(\hat{\mathbf{r}}_{cc}) (U_{\text{rel}} - V_C) \Psi_A d\mathbf{R}$$

$M_{-\eta, l+\frac{1}{2}}(2kr)$  is the “other” Whittaker function, irregular at  $r \rightarrow \infty$

## The laborious way (one open channel)

Set up as a particle in a box problem (i.e.  ${}^5\text{He}$  with box defined at 9 fm  $\alpha n$  separation)

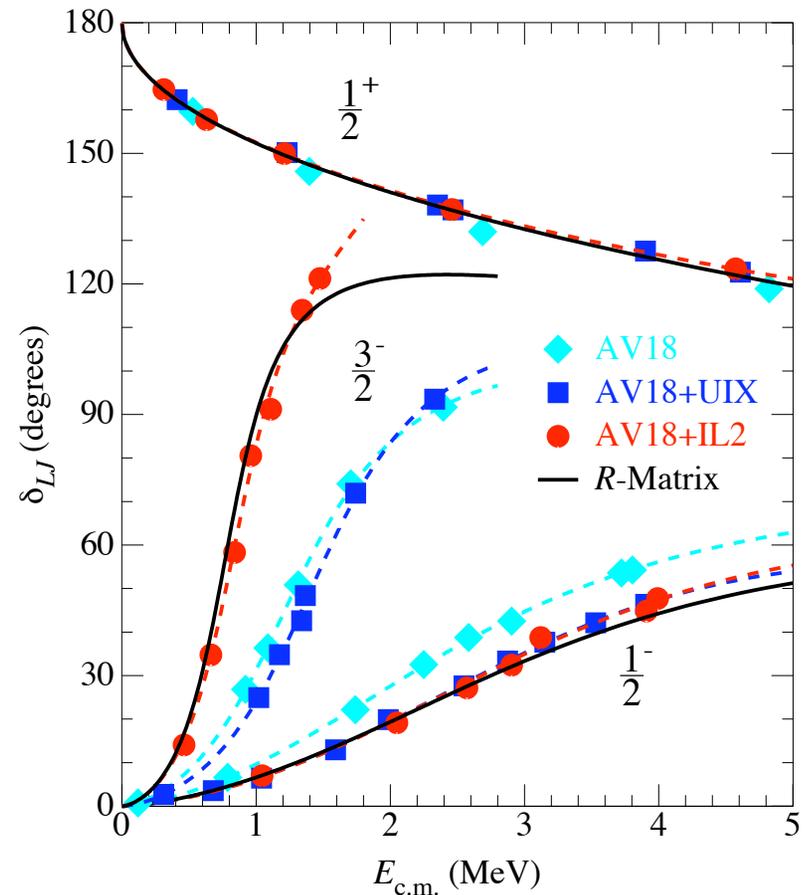
Specify a boundary condition  $\gamma = \psi'/\psi$  at the edge of the box

Compute energy of lowest state in the box

Match  $\psi \propto F_l \cos \delta + G_l \sin \delta$  across boundary

Get phase shift  $\delta$

Repeat for many  $\gamma$  to get  $\delta_i(E_i)$

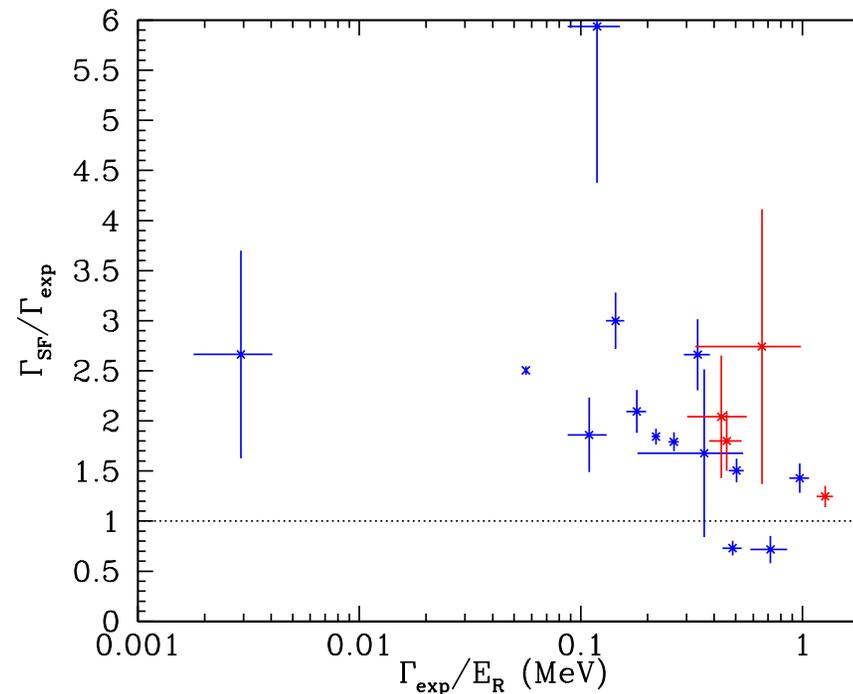


Nollett et al. PRL 99, 022502 (2007)

Fit  $\delta(E)$  to Padé form of  $S$ -matrix; width is imaginary part of pole energy

## Estimating the width from the VMC spectroscopic factor

The  $\theta^2 = S_{lj}$  estimate gets us within a factor of 3 of experiment in low-lying  $A \leq 9$  states



There is some ambiguity in the limit ( $\frac{3\hbar^2}{2\mu r^2}$  vs.  $\frac{\hbar^2}{\mu r^2}$  vs.  $\frac{(2l-1)\hbar^2}{(2l+1)\mu r^2}$  vs. Woods-Saxon single-particle width,  $r$  must be chosen)

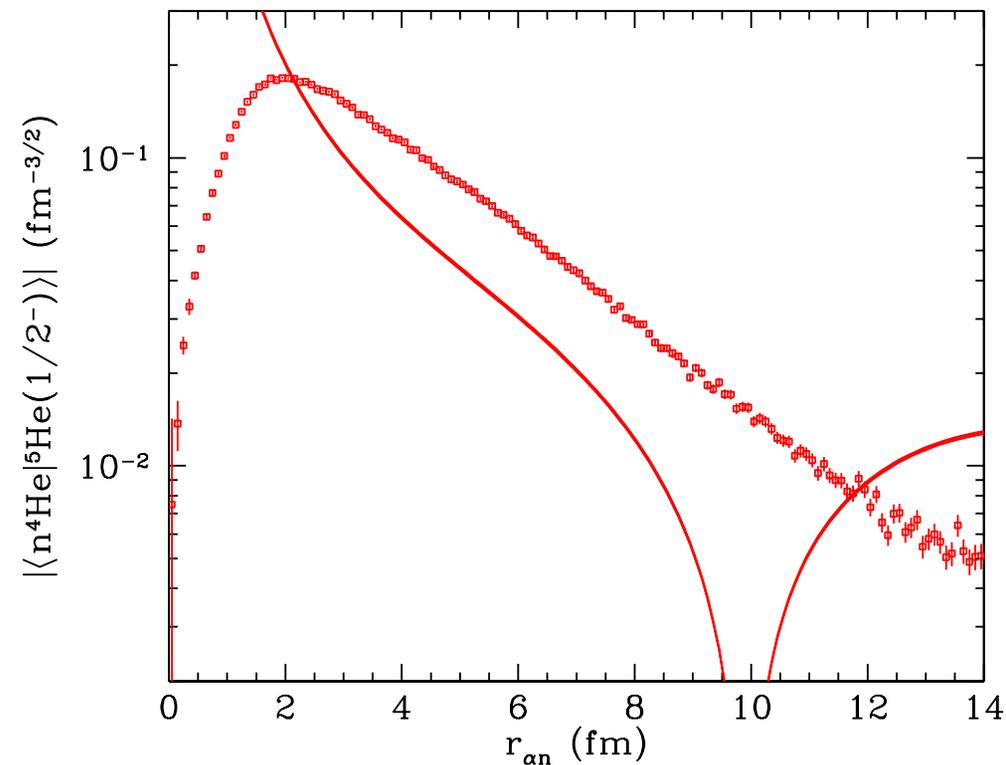
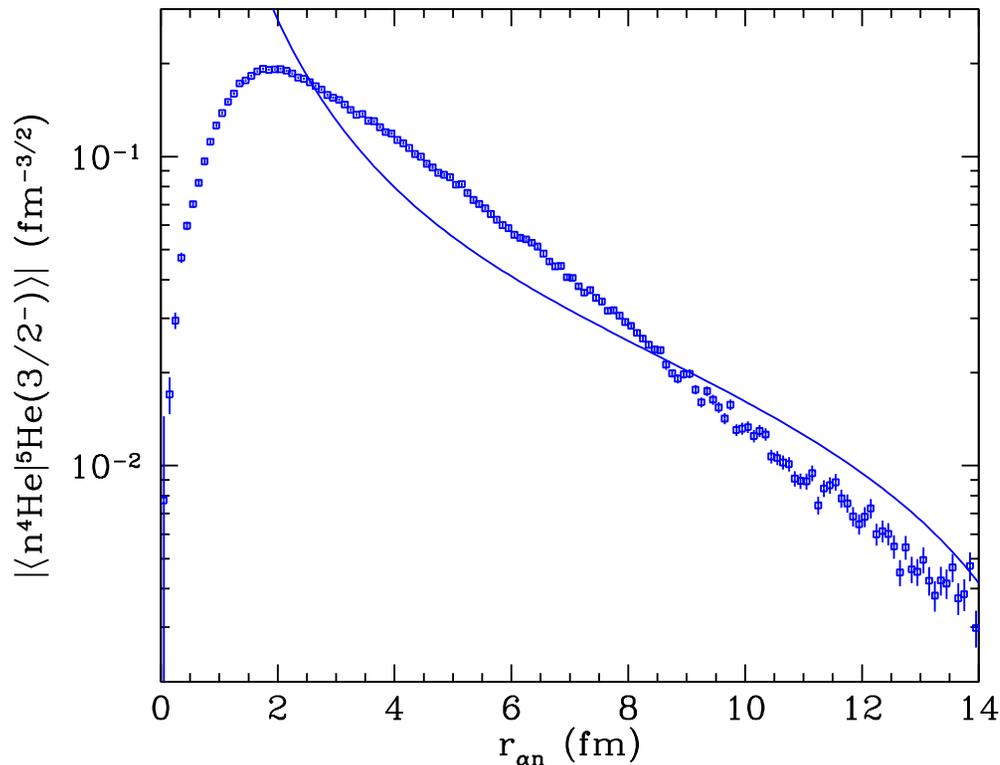
We could pick a definition that centers the trend and live with the scatter

## The good, the bad, the ugly

Lots of widths come out close to experiment

Widths not close to experiment generally have some unaccounted-for width (e.g.  $\alpha$  or 3-body channel) or isospin mixing ( $^8\text{Be}$   $3^+$  and  $1^+$  states), or are broad

Wiringa's pseudobound  $^5\text{He}$  states yield wildly unreasonable widths, probably because they're very broad



## $^8\text{Be } 3^+$ states

The first two  $3^+$  states are isospin-mixed ( $T = 0, 1$ )

The lower (19.07 MeV) is predominantly  $T = 1$ , upper (19.235 MeV) predominantly  $T = 0$

Computing them unmixed, I found 19.07 wider than 19.235, opposite from experiment

My first thought was that the isospin assignment might be backward

If I consider mixing, I can fit a mixing parameter to one observed width and predict the other width

Going either direction, I get a fair match for the predicted width, have to inflate errors a bit for true consistency

Mixing parameter comes out small relative to Barker 1978 (45 keV vs. 60 keV) and to GFMC (90 keV); direction is right but hard to judge agreement

## Pinkston-Satchler as a test of the 90° assumption

The integral relations I've been using are special cases of the overlaps of Pinkston & Satchler (or Kawai & Yazaki):

$$R_{lj}(r) \propto \left[ \cos \delta_{lj} + \int_r^\infty G_l(kr') \phi_1^\dagger \phi_2^\dagger (V - V_C) \Psi d\mathbf{R} \right] F_l(kr)/r \\ + \left[ \int_0^r F_l(kr') \phi_1^\dagger \phi_2^\dagger (V - V_C) \Psi d\mathbf{R} \right] G_l(kr)/r$$

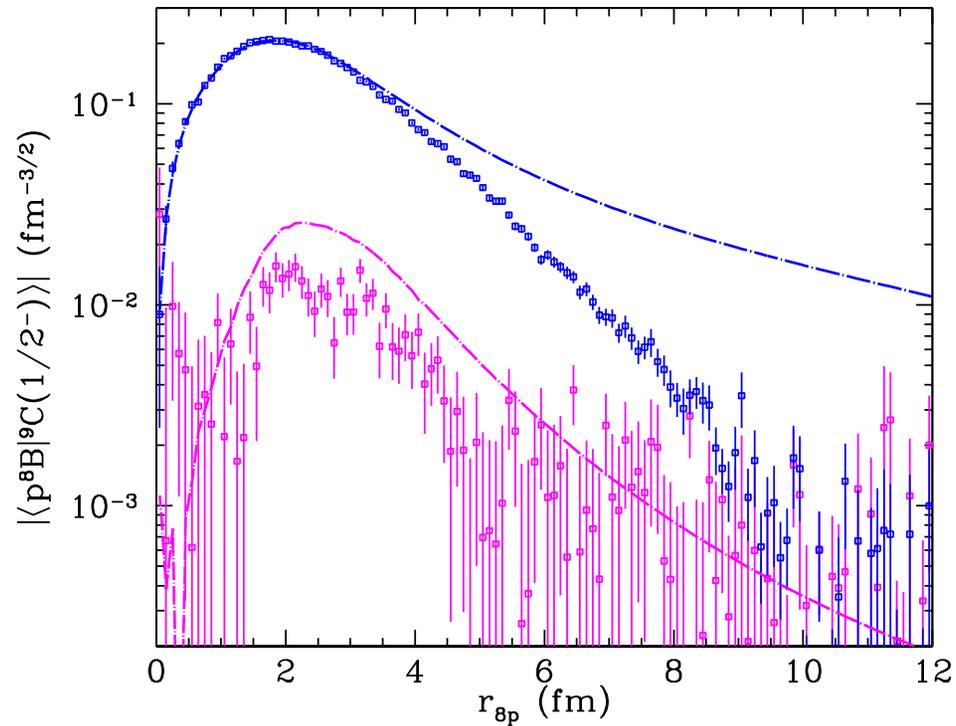
90° phase shift means no  $F_l$  component at  $r \rightarrow \infty$

If this  $R_{lj}$  with  $\cos \delta_{lj} = 0$  is a poor match to the directly-computed overlap at small  $r$ , then  $\delta \neq 90^\circ$  for that channel  $\rightarrow$  my assumptions are invalid

Cases that fail this test generally have small spectroscopic factors

## Pinkston-Satchler as a test of the $90^\circ$ assumption

Good

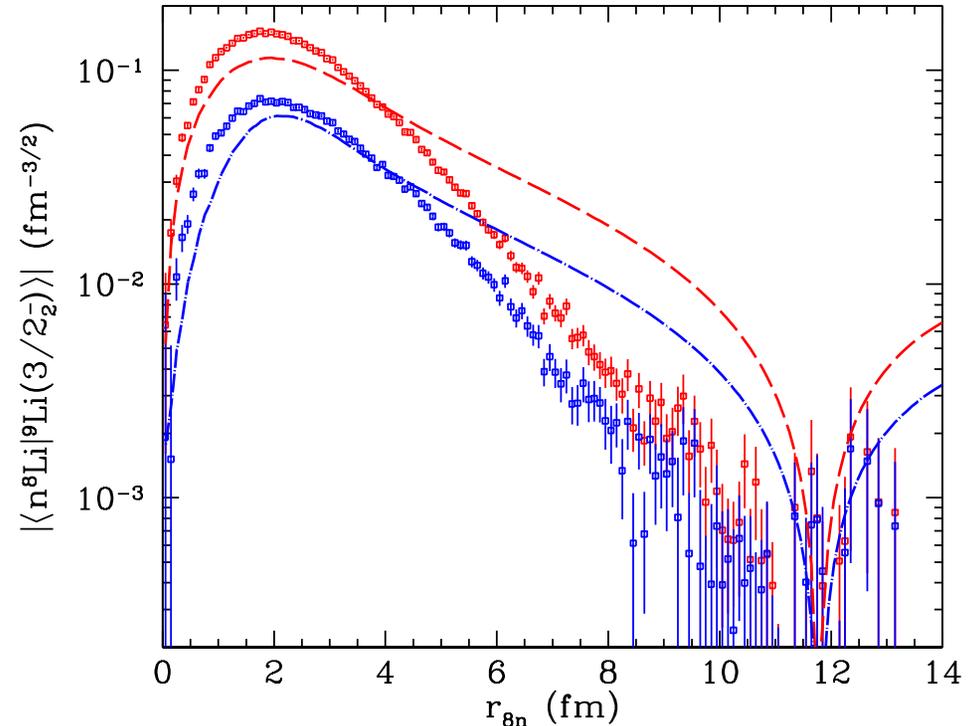


$p_{3/2}$

$p_{1/2}$

$f_{5/2}$

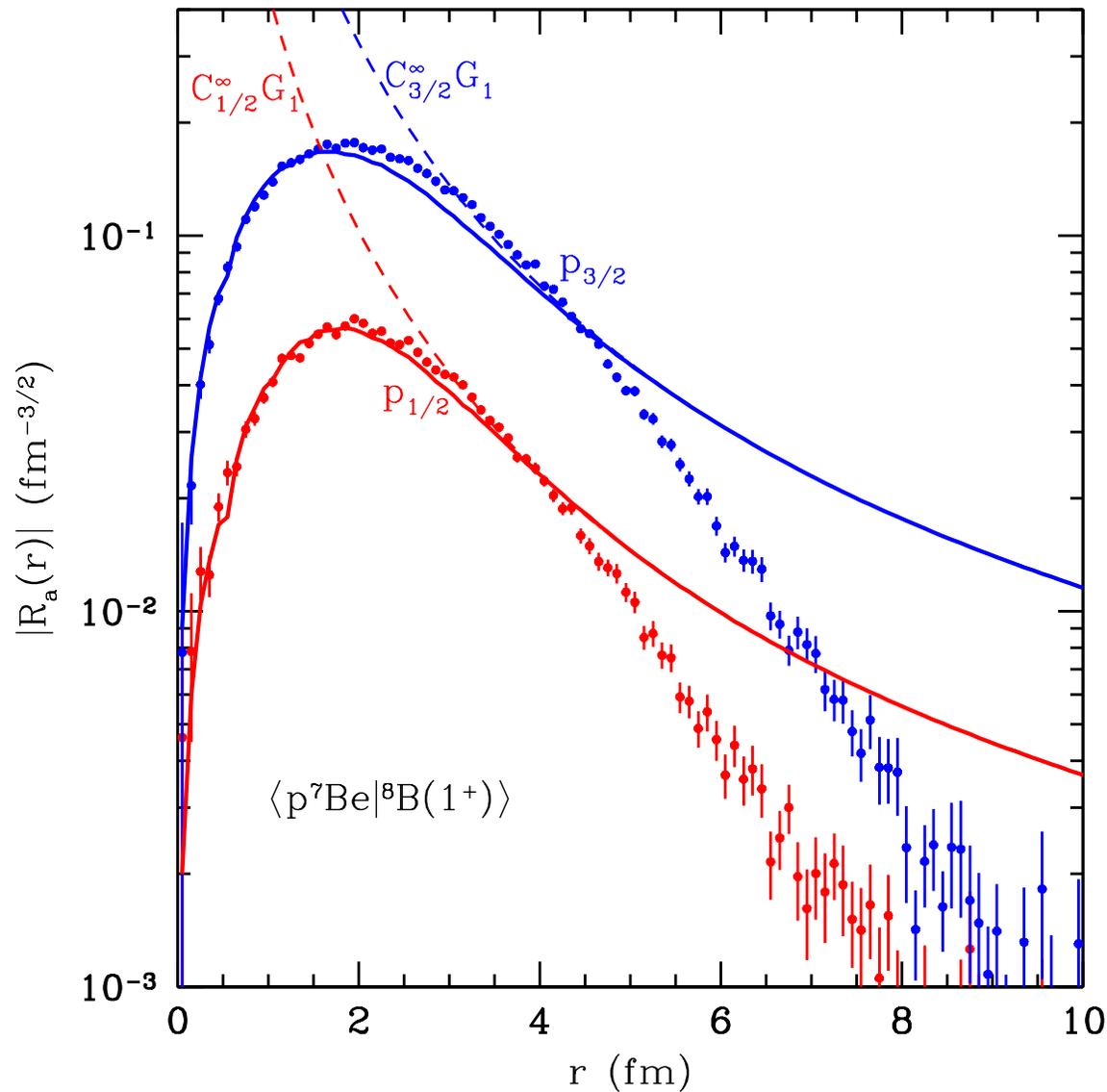
Bad



Fitting  $A$  to match VMC overlap is a bad idea: difference can be a shortcoming of VMC

## Overlaps at all radii

### Close-up of consistent overlaps



Integral and direct overlaps  
agree at  $r < 4$  fm

At large  $r$ , integral method  
obeys resonance  
asymptotics (by definition)

## ${}^7\text{He}$ and ${}^9\text{He}$ ; ${}^8\text{Li}$ & ${}^8\text{B}$ $0^+$ states

There was a hope that we could say something useful about widths & identification of states in  ${}^7\text{He}$  and  ${}^9\text{He}$

${}^7\text{He}(\frac{1}{2}^-)$  and  ${}^7\text{He}(\frac{5}{2}^-)$  both fail  $90^\circ$  consistency test and are rather broad

In  ${}^9\text{He}$ , broad  $\frac{1}{2}^-$  matches width claimed at Dubna (but not elsewhere)

Other  ${}^9\text{He}$  states ( $\frac{1}{2}^+$ ,  $\frac{3}{2}^-$ ,  $\frac{3}{2}^+$ ,  $\frac{5}{2}^+$ ) don't seem to have anything to do with  ${}^8\text{He}$  g.s. (not  $90^\circ$ , small spectroscopic factors)

$0^+$  states in  ${}^8\text{Li}$  and  ${}^8\text{B}$  come out broad and look unreliable –  $90^\circ$  consistency test fails, mismatch between integral & direct overlaps

## Uses of overlaps

The overlap functions can help to make reaction theory consistent with structure theory (breakup,  $(d, p)$ ,  $({}^3\text{He}, d)$ ...)

Several papers now use VMC overlaps (computed directly, not Pinkston-Satchler) as inputs via fitted Woods-Saxon wells:

$$(T + V_{WS})R_{lj} = ER_{lj}$$

with  $R_{lj}$  from VMC

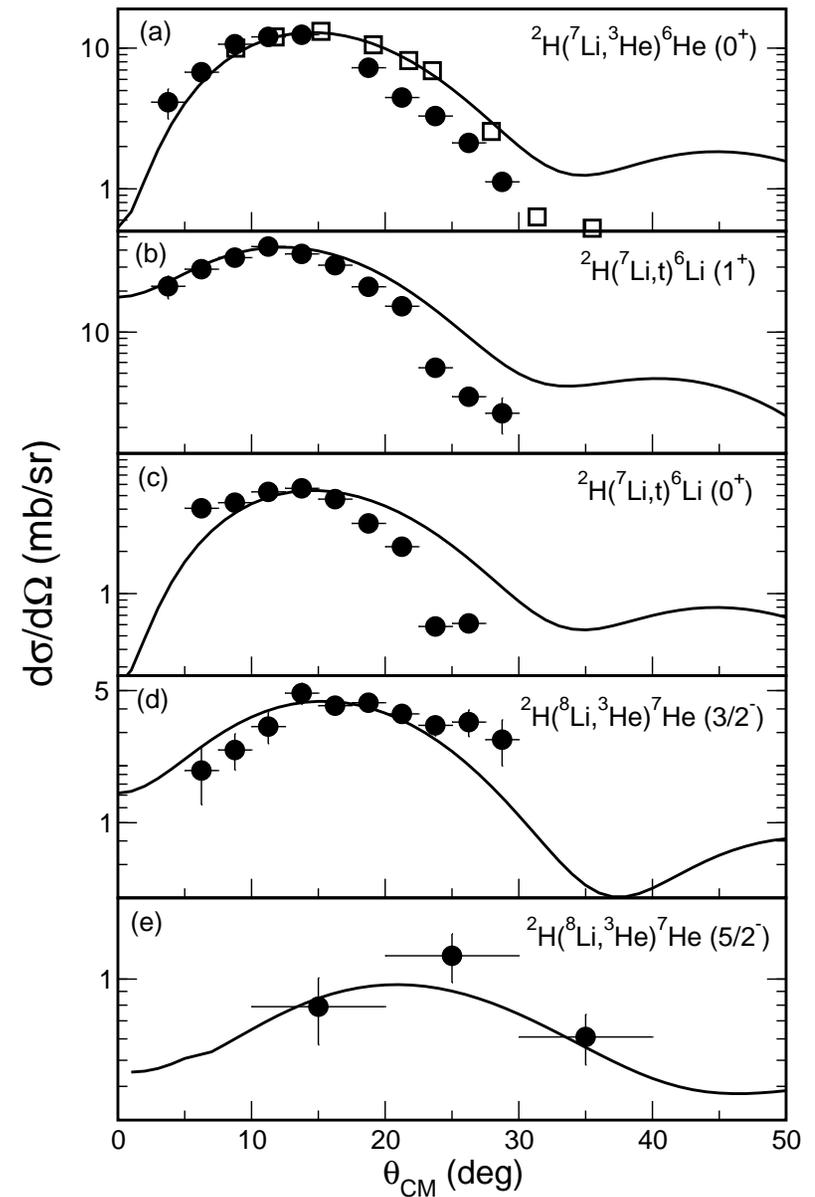
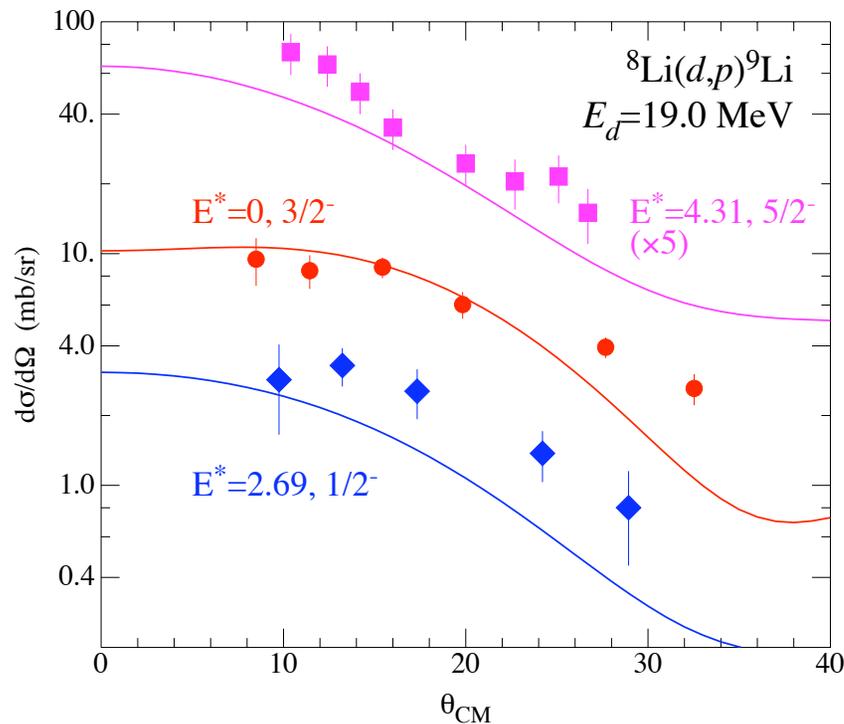
- Wuosmaa et al. PRC 72, 061301(R) (2005); PRL 94, 082502 (2005); PRC 78, 041302(R) (2008)
- Kanungo et al., PLB 660, 26 (2008)
- Grinyer et al. PRL 106, 162502 (2011)

With overlaps as input & no further fudging, experiment & VMC results agree (same spectroscopic factor – even for  ${}^7\text{Li}$ , *pace* ubiquitous graphs)

## Some examples

DWBA with  $\langle A - 1 | A \rangle$  vertices from VMC overlaps

There's still an optical potential, e.g. for  ${}^8\text{Li}+d$



## Integral method vs. factorized widths from spectroscopic factors

I could have always made rough estimates of widths using a factorization  $S_{lj}\Gamma_{\text{s.p.}}$ ,  
with  $S_{lj}$  from QMC

Wigner (causality) limit is easy to use for  $\Gamma_{\text{s.p.}}$  but not much good

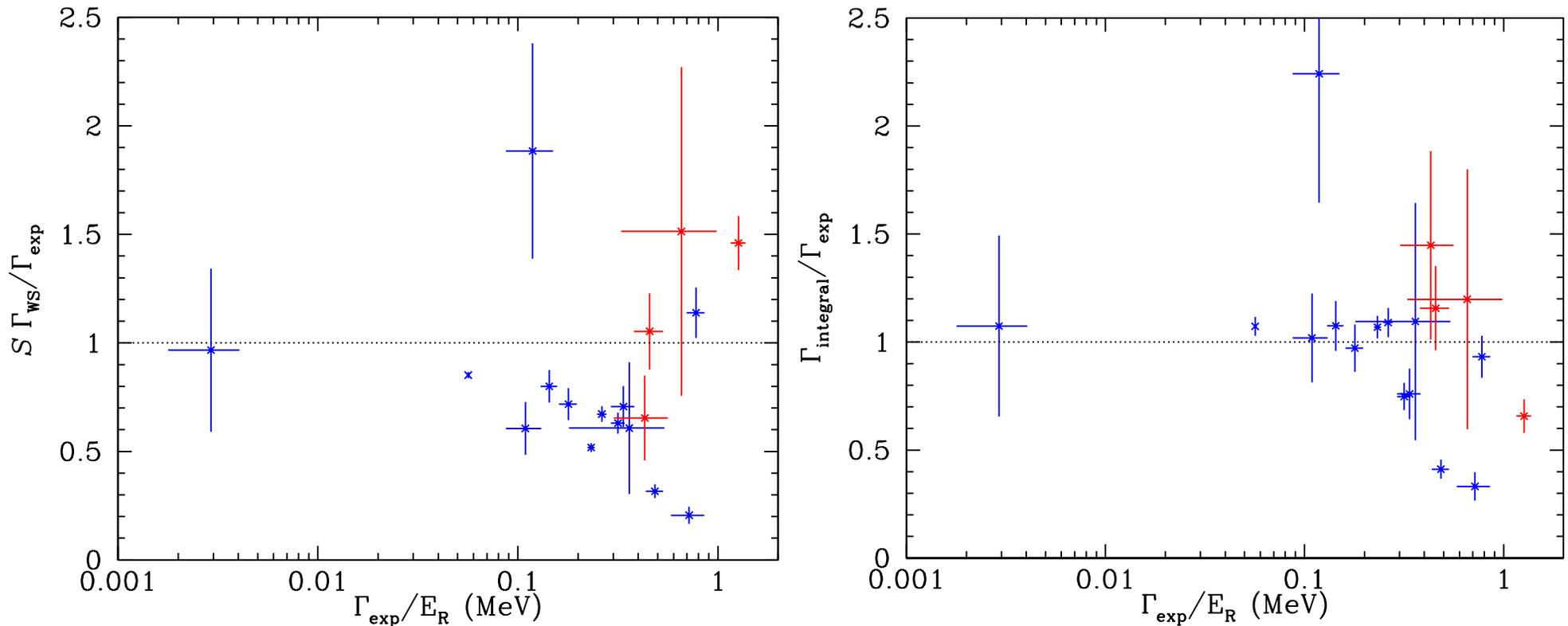
Shell-model studies often use “single-particle” widths computed from Woods-Saxon well

“Standard” geometric parameters are used & depth is set to match resonance energy, obtaining  $\Gamma_{\text{WS}}$

This width is multiplied by the spectroscopic factor:  $S_{lj}\Gamma_{\text{WS}}$

Geometric parameters should then be varied within “reasonable” bounds to check sensitivity

## Is the width integral better than the Woods-Saxon width times $S_{lj}$ ?



blue: consistent with  $90^\circ$  via P-S

red: not consistent

For narrow states without open  $\alpha$  channels, it's good and apparently an improvement

Mean of vertical axis, states where all channels counted & VMC wave function  
"looks resonant:"  $1.06 \pm 0.07$  integral,  $0.75 \pm 0.15$  Woods-Saxon

## ${}^7\text{He}$ and ${}^9\text{He}$ ; ${}^8\text{Li}$ & ${}^8\text{B}$ $0^+$ states

There was a hope that we could say something useful about widths & identification of states in  ${}^7\text{He}$  and  ${}^9\text{He}$

The  ${}^7\text{He}$  ground state comes out too narrow:  $\Gamma = 87(5)$  keV vs.  $\Gamma = 125_{-15}^{+40}$

${}^7\text{He}(\frac{1}{2}^-)$  and  ${}^7\text{He}(\frac{5}{2}^-)$  both fail  $90^\circ$  test and are rather broad;  $\frac{1}{2}^-$  width isn't bad

In  ${}^9\text{He}$ , broad  $\frac{1}{2}^-$  matches width claimed at Dubna (but not elsewhere)

Other  ${}^9\text{He}$  states ( $\frac{1}{2}^+$ ,  $\frac{3}{2}^-$ ,  $\frac{3}{2}^+$ ,  $\frac{5}{2}^+$ ) don't seem to have anything to do with  ${}^8\text{He}$  g.s. (not  $90^\circ$ , small spectroscopic factors)

Stopped there to avoid unbound decay products (so no decays through  ${}^8\text{He}(2^+)$ )

Calculations of broad unobserved(?)  $0^+$  states in  ${}^8\text{Li}$  and  ${}^8\text{B}$  look unreliable –  $90^\circ$  test failed, looks like  ${}^5\text{He}$

## Nuclear widths, real?

Comparison with experiment still needs some work

What I extract should (I think) be identified with the “formal width”  $\Gamma(E_R)$

But since what is observed generally looks like

$$\frac{d\sigma}{d\Omega}(E) \propto \frac{\Gamma^2(E)}{\left[E - E_\lambda - \sum_c \gamma_c^2 S_c(E)\right]^2 + \frac{1}{4}\Gamma^2(E)},$$

an experimental FWHM looks something like

$$\Gamma_{\text{obs}} = \frac{\Gamma(E_R)}{1 + \sum_c \gamma_c^2 S'_c(E_R)}$$

Digging back into the TUNL and Ajzenberg-Selove compilations has been sobering

It's sometimes hard to tell which flavor a quoted  $\Gamma$  has

Sometimes tabulated  $\Gamma$  is average of  $\Gamma$  &  $\Gamma_{\text{obs}}$  from different experiments

Original references don't always inspire confidence